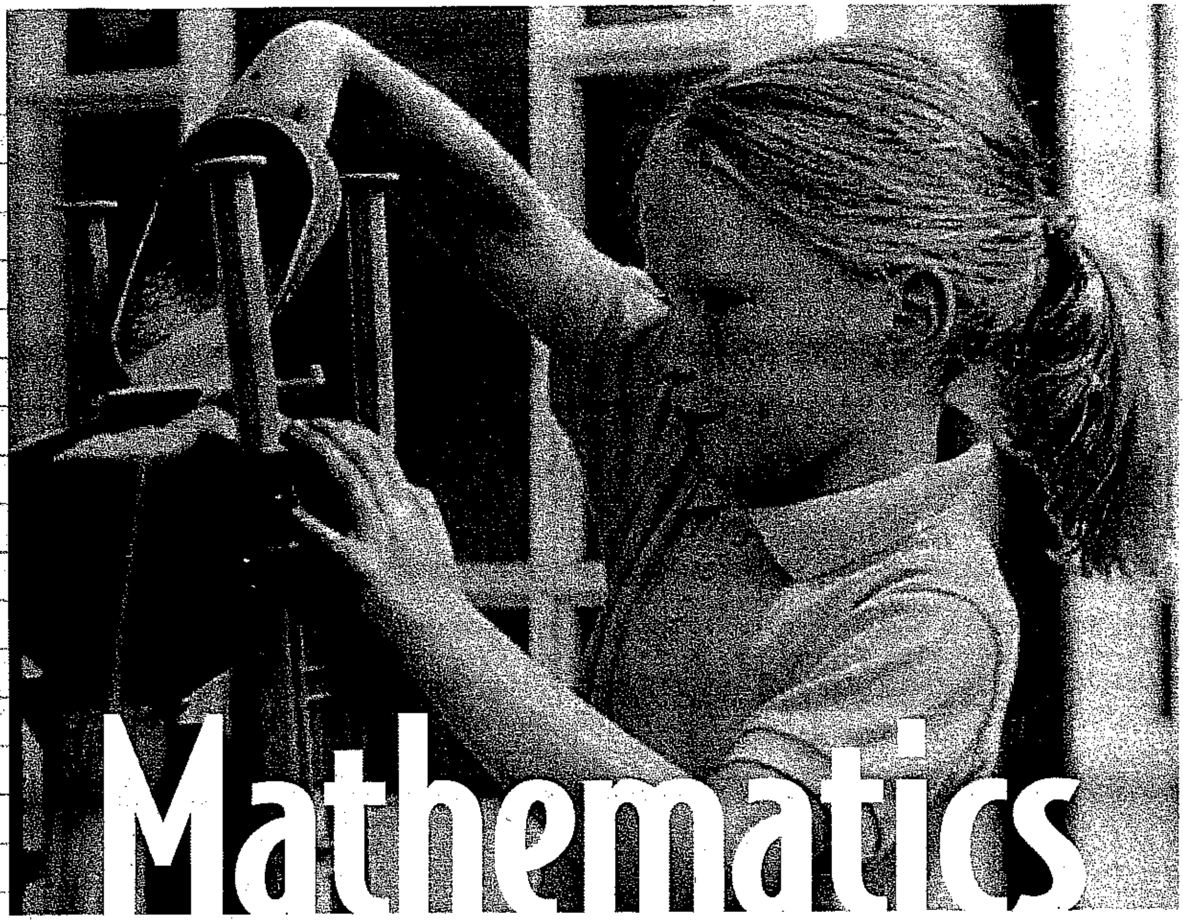

Mathematics in the Preschool

Douglas H. Clements

"Anyone who is pushing arithmetic onto preschoolers is wrong. Do not hurry children. No math in preschool!"

"What else is preschool for if teachers do not get children ready for school? They should teach the children basic skills and how to sit and listen."

Principles and Standards for School Mathematics identifies a new age band that includes preschoolers for the first time (NCTM 2000). What mathematics instruction is appropriate for these young children? The two speakers have different opinions. I think that each is a little bit right and a little bit wrong.

High-quality teaching in mathematics is about challenge and joy, not imposition and pressure. Good early childhood mathematics is broader and deeper than mere practice in counting and adding. It includes debating which child is bigger and drawing maps to the "treasure" buried outside. Quality mathematics instruction includes providing loads of unit blocks, along with loads of time to use them; asking children to get just enough pencils for everyone in the group; and challenging children to estimate and check how many steps are required to walk to the playground (see **fig. 1**).

Much of our world can be better understood with mathematics. Preschool is a good time for children to become interested in counting, sorting, building shapes, finding patterns, measuring, and estimating. Quality preschool mathematics is not elementary arithmetic pushed onto younger children. Instead, it invites children to experience mathematics as they play in, describe, and think about their world.

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Do We Really Need *Preschool* Mathematics?

We need preschool mathematics for four reasons.

First, preschoolers already experience curricula that include only a small amount of mathematics—and usually that content is anemic. We should improve this situation.

Second, many of these children, especially those from minority and low-income groups, later experience considerable difficulty in school mathematics. Recent curriculum development projects have shown that the gap between these and other children can be narrowed. We should address these equity issues.

Third, preschoolers possess informal mathematical abilities and enjoy using them. Before they enter school, many children develop number and geometry abilities that range from counting objects accurately, to finding one's way through the environment, to making shapes. Children use mathematical ideas in everyday life and develop informal mathematical knowledge that is surprisingly complex and sophisticated. Neglecting to nurture such interests would be an educational shame.

For example, five-year-old Chris is making shapes with a simplified version of Logo software. He has been typing R (for rectangle), then two numbers for the lengths of the sides. This time he chooses 9 and 9. He sees a square and laughs. A nearby adult asks, "Now, what do the two 9s mean for the rectangle?" Chris replies, "I don't know, now! Maybe I'll name this a square rectangle!" Chris uses his invented terminology repeatedly on succeeding days. Similarly, with the concept of number, children as young as three years old understand the basic principles of counting, even as they work to polish their skills.

Finally, although recent research on the brain has less to tell us about education than some suppose, it offers three general messages: (1) Preschoolers' brains undergo significant development, (2) preschoolers' experience and learning affect the structure and organization of their brains, and (3) preschoolers' brains grow most as the result of complex activities, not from simple skill learning.

Consider Alex, who just turned five and whose brother, Paul, is three years old. She wandered into the room and made an announcement:

Alex. When Paul is six, I'll be eight; when Paul is nine, I'll be eleven; when Paul is twelve, I'll be fourteen . . . [she continues until Paul will be eighteen and she will be twenty].

Adult. My word! How on earth did you figure all that out?

Alex. It's easy. You just go "three-FOUR-five" [clapping on the four]; you go "six-SEVEN-eight"; you go "nine-TEN-eleven."

Alex put together two aspects of her experience: counting and songs that she sang rhythmically while jumping rope. This approach made sense to her, far more so than if an adult had tried to teach her an "add 2" algorithm.

We see that preschoolers, including those from minority and low-income groups, are competent in informal mathematics, and they show spontaneous interest in "big" mathematical ideas. If preschoolers internalize informal mathematics, why are so many at risk for failure in later mathematics learning? The main reason may be that they do not have the support required to build connections with school mathematics. They are self-motivated to investigate patterns, shapes, measurement, the meaning of numbers, and how numbers work, but they need assistance to bring these ideas to an explicit level of awareness. Such awareness is an essential component of mathematical knowledge.

High-quality teaching in mathematics is about challenge and joy, not imposition and pressure

Preschoolers and Mathematics

People of all ages actively construct mathematical knowledge, but preschoolers are a special group, and we need to plan their instruction with care. Con-

FIGURE 1 Children deal with length, horizontal lines, parallelism, and symmetry in their block building.

FIGURE 2

Measuring the perimeter of a rug

sider two of their special characteristics.

First, the ideas that preschoolers construct can be quite different from those of adults. Preschool teachers must be particularly careful not to assume that children "see" situations, problems, or solutions as adults do. For example, one researcher asked Brenda to count six marbles. Then the researcher covered them up, showed one more, and asked how many he had in all. Brenda said that he had one. When the researcher pointed out that he had six marbles hidden, Brenda said adamantly, "I don't see no six!" For Brenda, no number could exist without objects to count.

Successful teachers interpret what a child is doing and thinking and attempt to see the situation from the child's point of view. From their interpretations, these teachers speculate about what concepts the child might be able to learn or abstract from his or her experiences. Similarly, when teachers interact with children, they should also consider their own actions from the children's points of view. This need for interpretation and conjecture makes early childhood teaching both demanding and rewarding. Brenda's teacher, for example, might hide four marbles, then encourage Brenda to put up four fingers to represent the hidden marbles.

A second characteristic of young children is that they do not perceive or act on their world as if it were divided into separate subjects. Successful preschool teachers help children develop premathematical and mathematical knowledge throughout the day. They plan activities that simultaneously promote intellectual, social, emotional, and physical development. When preschoolers do mathematics, they really *do* it—acting with their whole beings. For this reason, discussions of symmetry that use children's own vocabulary should be as common in the block corner and at the art easel as in any formal mathematics activity.

Such holistic teaching and learning capitalizes on preschoolers' high level of motivation to learn in a self-directed manner. This teaching promotes a view of mathematics as a positive, self-motivated, self-directed problem-solving activity at the time that children first develop their mathematical beliefs, habits, and feelings.

Children's play and interests are the sources of their first mathematical experiences. These experiences become mathematical as the children represent and reflect on them. Young children represent their ideas by talking but also through models, dramatizations, and art. From the motor and singsong beginnings of pat-a-cake stem the geometric patterns of a "fence" built from unit blocks and the gradual generalization and abstraction of patterns throughout the child's day: "See, my drawing uses the same pattern as your blocks did!"

The Teacher's Role

High-quality learning is often incidental and informal but not unplanned or unsystematic. The teacher's role in fostering this learning is complex. Consider the teacher's many responsibilities for mathematics and free play. The teacher must plan an environment that is conducive to mathematical explorations; for example, the environment should include unit blocks, a shopping center, and manipulatives. Their play with such objects forms much of the premathematical conceptual foundation that children need.

To help children build on this foundation for developing mathematical knowledge, teachers must observe children and intervene when necessary. But when is intervention necessary? A useful strategy is to ask whether mathematical thinking is *developing* or whether it is *stalled*. If it is developing, the teacher might observe and take notes, leave the children alone, and talk about the experience later with the children or the whole class to explicate the mathematics.

One teacher heard two girls arguing about who had the taller block tower. She observed them as they compared the height of their towers with their bodies. Later, she asked them to explain to the class what they had done. The children shared other experiences that they had had in comparing how tall objects were, as well as different ways in which they could measure height, such as by counting units or by using direct or indirect comparison.

In contrast, when mathematical thinking is stalled, the teacher might intervene, discussing and clarifying the ideas. For example, if two children argue about whose block building is "bigger," the teacher might see that one child is talking about height and that one is talking about width, area, or volume. This teacher might remark that the buildings are big in different ways: "Colleen has a very tall building, and Chris's seems to be very wide." She might ask the children what they see and, later, discuss the issue with the class as an interesting event. Other interventions are subtle. When observing children comparing the length of two rugs, a teacher may quietly ensure the accessibility of connecting cubes, string, and other objects that might be used for measuring (see **fig. 2**).

Teachers might also work with children to develop such interests into a full project. An example of such a project emerged at one of the famous preschools in Reggio Emilia, Italy. *Shoe and Meter* (Malaguzzi 1997) tells the full story of the project, using the teacher's own photographs, but the summary is as follows: A group of children wants another work table just like one they already have. A local carpenter says that he will build the table, but he needs measurements. The children first try to measure with their fingers. Two children leave and return with paper, saying, "We have to draw the table so we can understand it." The children then return to measuring so that they can label their drawings. They try to measure using heads, fists, hand spans, and legs. They seem to believe that a longer unit will economize their work. The children then turn to objects, such as books. They seem to realize that using objects will be easier than using body parts.

Eventually, after working on other measurement tasks suggested by their teachers, the children realize that they need a type of measurement that they can share. They start by making their own rulers, which is the first occasion that they have had to write numbers for their measures. One child is dissatisfied with having only numerals on the ruler, and the children begin to separate the numerals with lines. The lines help them see the need for equal distance between one number and the next. The children all make their own rulers, but confusion reigns when they first use them—the children get measurements of 78, 41, 20, and so forth. They all laugh uproariously. The teacher asks the children to line up their rulers. The children shout that they must pick the one with the "right numbers."

Next, the children make new rulers but, surprisingly, also return to measuring with objects. They use a shoe, stepping it off on a strip of paper that they place on the tabletop to mark the distances. They measure in the other direction and are sure that they are ready to give measurements to the carpenter, because the shoe always yields the same measure! They draw their results carefully, then one child suggests getting a "real" meterstick. They do so and measure . . . the shoe! It is 20 centimeters long. They use a calculator to add 20 for each shoe length, then write both measures on their drawing of the tabletop. After additional work, they make a final drawing and give it to the carpenter, along with a letter encouraging him to "do a good job."

The Reggio school activity is a remarkable educational experience, capturing the depth of children's engagement and learning. Teachers observe the activity, document children's progress, and intervene with care. Children represent their thinking to great benefit. The activity suggests that myriad projects can emerge from children's interests and contribute to their development in mathematics.

By their nature, projects will vary, as will chil-

FIGURE 3

In the Building Blocks software, children must use specific tools—slides, flips, and turns—to make a picture. This illustration shows the use of the "turn" tool. Activities such as these help children become explicitly aware of geometric motions.

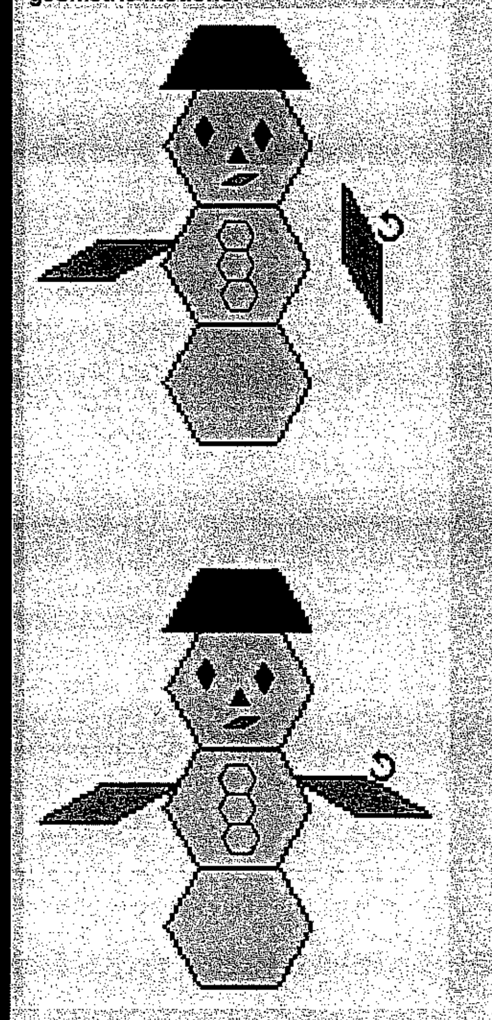
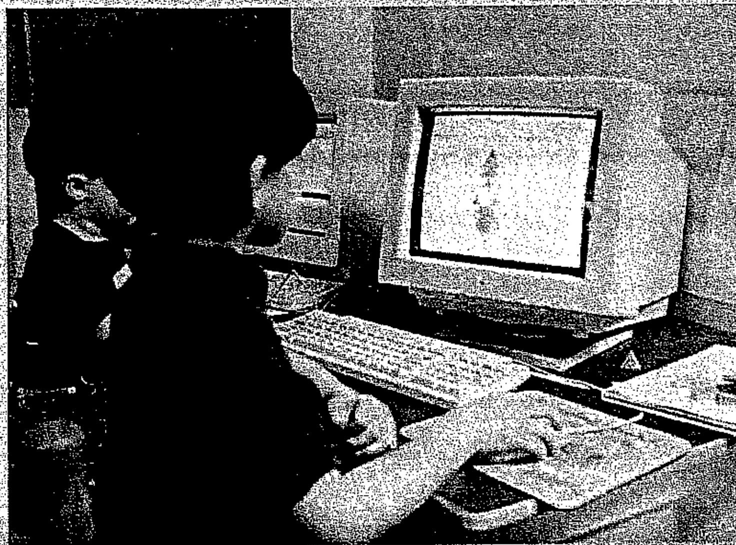


FIGURE 4

Children build designs with pattern blocks and copy their designs at the computer.



dren's thinking as they work on projects. Many older children will continue to struggle with measurement ideas and skills. Research indicates that traditional assumptions used in teaching measurement may be faulty, and new approaches may tap unrealized potential in young children. Most important, projects allow children of all levels of readiness to become involved meaningfully with mathematics.

Another obvious way to develop mathematical knowledge is to plan and introduce activities that specifically deal with mathematics. For example, games that use number cards, such as war, or board games with number cubes provide experiences with counting and comparison. Kamii and Housman (1999) present numerous examples. In addition, many children's books have mathematical themes. Problem-solving number activities can be successful even with three- and four-year-old children, developing counting and number abilities along with such reasoning capacities as classifying and ordering. Recent work confirms that appropriate curricula strengthen preschoolers' number and geometric knowledge. In all these settings, the teacher enhances children's mathematical ideas and vocabulary by posing such questions as "Did you try this way?" "What would have happened if . . . ?" "Do you think that you could . . . ?"

One final way to develop mathematical knowledge is through ongoing use of appropriate technology. For example, an activity in the Building Blocks software package asks children to build a picture on the computer using specific tools for geometric motions (see **fig. 3**). Note that the child also used the "size" tool to add facial features and buttons that she could not make with actual pattern blocks. Because each motion must be chosen deliberately on the computer, children often avoid turning and flipping shapes. A good teaching strategy is to have them make a design with physical blocks, then copy it onto the computer (see **fig. 4**). In this way, they perform many flips and turns and they connect the physical activity with the computer task.

Conclusions

Successful preschool teachers build on children's everyday activities, incorporating their cultural backgrounds, languages, and mathematical ideas and strategies. They use a variety of instructional strategies, create meaningful child-related contexts, and offer opportunities for active participation, to help children learn premathematical and mathematical ideas and develop positive beliefs about mathematics and themselves as budding mathematicians.

NCTM's *Principal and Standards for School Mathematics* provides guidelines for the topics to be taught in mathematics. We must remember, though, that how we teach mathematics is just as important as the topics we teach. The most powerful mathematics for a preschooler is usually not acquired while sitting down in a group lesson but is brought forth by the teacher from the child's own self-directed, intrinsically motivated activity.

Preschoolers can and should engage in mathematical thinking. All young children possess informal mathematics and can learn more. Teachers should build on and extend the mathematics that arises in children's daily activities, interests, and questions. They should struggle to see children's points of view and use their interpretations to plan their interactions with children and the curriculum. This approach ensures that mathematical content will be meaningful for young children. A combination of an environment that is conducive to mathematical explorations, appropriate observations and interventions, and specific mathematical activities helps preschoolers build premathematical and explicit mathematical knowledge.

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