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Introduction

Mission Statement

The Mississippi Department of Education (MDE) is dedicated to student success, including the improvement of student achievement in English Language Arts (ELA) and mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a global society. The Mississippi College- and Career-Readiness Standards (MS CCRS) provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level or course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy. The goal of the MDE is to provide educators with the training and resources to understand and implement the MS CCRS effectively.

Purpose

In efforts to facilitate implementation and promote understanding of the MS CCRS for ELA and mathematics, the W. K. Kellogg Foundation generously awarded the MDE a grant to secure a cadre of effective educators to develop the MS CCRS Exemplar Units for teachers. Specifically, a group of highly-effective Mississippi educators developed exemplar instructional units and lessons aligned to the MS CCRS for ELA and mathematics. The MS CCRS Exemplar Units address difficult-to-teach standards as determined by teachers and are designed to serve as exemplar models for instructional units, lessons, and resources. The MS CCRS Exemplar Units have been vetted through nationally renowned vendors to ensure exemplar quality.
Design Overview

The MS CCRS Exemplar Units for ELA and mathematics address grade-level specific standards for Pre-Kindergarten-8\textsuperscript{th} grade, as well as for Algebra, English I, and English II. The overall unit plan is described in the first section of the ELA and math units. This section includes the unit title, a suggested time frame, the grade level MS CCRS addressed and assessed, a unit overview with essential questions and a summary of lesson tasks, and the culminating/performance task description and rubric.

Though the math and ELA overall unit plan designs are very similar, some design aspects differ in order to accommodate the respective requirements of each content area. For mathematics, the first section also provides a segment designated for the Standards for Mathematical Practices (SMPs) addressed in the unit. For ELA, the first section also includes a text set with links to texts (if in the public domain) and a fresh/cold-read task.

The second section of each unit includes lesson plans. Within the lesson plans, provided are lesson-specific MS CCRS, suggested time frames, learning targets, guiding questions, required resources and materials, vocabulary terms and instructional strategies, teacher directions, instructional supports for students, enrichment activities, student handouts, assessments (formative, summative, pre-, and self-), and additional resources to aid in the implementation of the lessons.

Implementation

The intention of the MS CCRS Exemplar Units for ELA and mathematics is to provide educators with resources to understand and implement the MS CCRS effectively. The implementation of the MS CCRS Exemplar Units for ELA and mathematics is voluntary. Additionally, the MDE will provide ongoing support for implementation of the MS CCRS Exemplar Units with initial regional trainings followed by site-specific support through our regional service delivery model. For regional and site-specific training, please contact the MDE Office of Professional Development.
<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Unit Title</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Ratios and Proportions</td>
<td>7 days 10 days</td>
</tr>
</tbody>
</table>

### Mississippi College- and Career-Readiness Standards for Mathematics

**Focus:**

**7.RP.2** Recognize and represent proportional relationships between quantities.

- **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **c.** Represent proportional relationships by equations. For example, if total cost, \( t \), is proportional to the number, \( n \), of items purchased at a constant price, \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).
- **d.** Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \( r \) is the unit rate.

**Additional:**

**7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks \( \frac{1}{2} \) mile in each \( \frac{1}{4} \) hour, compute the unit rate as the complex fraction \( \frac{1/2}{1/4} \) miles per hour, equivalently 2 miles per hour.*

### Standards for Mathematical Practice

- **SMP.1** Make sense of problems and persevere in solving them.
- **SMP.2** Reason abstractly and quantitatively.
- **SMP.3** Construct viable arguments and critique the reasoning of others.
- **SMP.4** Model with mathematics.
- **SMP.5** Use appropriate tools and manipulatives strategically.
- **SMP.6** Attend to precision.
- **SMP.7** Look for and make use of structure.
- **SMP.8** Look for and express regularity in repeated reasoning.
Unit Overview

Throughout this unit, students will expand their understanding of ratios. A major focus will be placed on the exploration of the characteristics of proportionality, specifically with an emphasis of developing academic vocabulary and connecting prior understanding. As students interpret tables, graphs, equations, diagrams, and verbal descriptions, they will discover multiple methods for finding the constant of proportionality. Students will write equations and solve for unknown variables, and as a result of the instructional activities in this unit, they will develop a strong understanding of interpreting and comparing the different representations and solving problems centered around these representations.

Essential Questions:
- How are proportional relationships used to solve real-world and mathematical problems?
- How is the constant of proportionality shown in multiple representations of proportional relationships?

Lesson Tasks

Lesson 1: Tables as Tools
Students will activate prior knowledge on ratios and proportions. Students will use tables to solve proportions and begin to develop academic vocabulary for the unit.

Lesson 2: Proportionality in a Table
Students will determine whether two quantities are in a proportional relationship by identifying equivalent ratios in a table. Students will identify the constant of proportionality (unit rate) in tables and verbal descriptions of proportional relationships.

Lesson 3: Comparing Tables and Graphs
Students will discover what makes two quantities proportional and begin to recognize proportional relationships in tables and graphs.

Lesson 4: Multiple Representations
Students create a proportional scenario and create different representations of the situation. Students will complete Reynaldo’s Trip to reinforce the ability to compare multiple representations of proportional relationships.

Lesson 5: Solving Proportional Relationships
Students will work collaboratively to solve problems using equivalent ratios. Students will strengthen their understanding of multiple strategies, including the use of linear graphs.

Lesson 6: Performance Task
Students will complete their Graffiti Wall and work with a partner on the Pump It Up performance task, which involves monitoring heart rate and comparing unit rates after performing various exercises.

**Performance/Culminating Task**

**Pump It Up!**
This task connects unit rate and proportionality to a real-world application of finding heart rate. During this activity, students calculate their heart rate after completing various physical activities. Students will analyze their data using tables, graphs, and equations as well as the heart rates of others. After examining the data, they will apply reasoning when responding to questions using the multiple representations as they pertain to unit rate and proportionality.

**Standard(s) Assessed:** 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d
**Student Rubric for Performance/Culminating Task**

<table>
<thead>
<tr>
<th>Novice (1)</th>
<th>Tables</th>
<th>Graphs</th>
<th>Equations</th>
<th>Reasoning and Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) I created 1-2 tables.</td>
<td>a) I graphed 1-2 situations.</td>
<td>a) I wrote 1-2 equations.</td>
<td>a) I provided little explanation.</td>
<td></td>
</tr>
</tbody>
</table>
| Apprentice (2) | a) I created 3 tables.  
b) The tables had minimal errors. | a) I graphed 3 situations.  
b) The graph was scaled correctly but difficult to read. | a) I wrote 3 equations.  
b) I showed little evidence of how to use an equation while answering questions. |
| Practitioner (3) | a) I created 4 tables  
b) The tables had minimal errors.  
c) The tables were clear and precise. | a) I graphed 4 situations.  
b) The graph was scaled correctly.  
c) I used my graph correctly when answering questions about coordinates. | a) I wrote 4 equations.  
b) I used equations to support my answers.  
c) I used equations to correctly answer unit rate questions. |
| Expert (4) | a) I created 5 tables.  
b) The tables had no mistakes.  
c) Tables were clear and precise.  
d) I used the coordinates from the table to correctly make ratios and justify answers. | a) I graphed 5 situations.  
b) My graph was scaled correctly.  
c) I used my graph correctly when answering questions about coordinates.  
d) I used my graph correctly when comparing unit rates to determine proportionality. | a) I wrote 5 equations.  
b) I used equations to correctly justify answers.  
c) I used equations to correctly find unit rate.  
d) I correctly solved equations when given values for the variable. |

Lesson 1: Table as a Tool

Focus Standard(s): 7.RP.2a, 7.RP.2b
Additional Standard(s): 7.RP.1
Standards for Mathematical Practice: SMP.1, SMP.3, SMP.7
Estimated Time: 50 minutes

Resources and Materials:
- Butcher Paper
- Markers
- Handout 1.1: Graffiti Wall
- Handout 1.2: Table as a Tool
- Unit Rates Video: https://www.youtube.com/watch?v=SpZQFKU5P70

Lesson Target(s):
- Students will activate prior knowledge of ratios and proportions.
- Students will use tables to solve proportions and begin to develop academic vocabulary pertaining to ratios and proportions.

Guiding Question(s):
- What tools can be used to help find unit rate?
- How is unit rate used to determine proportionality?

Vocabulary

<table>
<thead>
<tr>
<th>Academic Vocabulary:</th>
<th>Instructional Strategies for Academic Vocabulary:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Rate</td>
<td>□ Introduce words with student-friendly definitions and pictures</td>
</tr>
<tr>
<td>Constant Rate</td>
<td>□ Model how to use the words in discussion</td>
</tr>
<tr>
<td>Proportional</td>
<td>□ Create pictures/symbols to represent words</td>
</tr>
</tbody>
</table>
• Table of Values
• Rate

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type of Text and Interpretation of Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level</td>
</tr>
<tr>
<td>✓</td>
<td>Assessment (Pre-assessment, Formative, Self, or Summative)</td>
</tr>
</tbody>
</table>

**Instructional Plan**

**Understanding Lesson Purpose and Student Outcomes:** Students will be introduced to how unit rate relates to proportionality. Students will find the unit rate from a table and complete tables with missing values using the unit rate. Students will compare different methods and decide which method they prefer.

**Anticipatory Set/Introduction to the Lesson: Graffiti Wall**

Distribute **Handout 1.1: Graffiti Wall**. This activity will require students to recall prior knowledge about ratios and proportions. Instruct students to brainstorm and record their ideas (e.g., terms, examples, pictures) onto the Graffiti Wall and share ideas with their team. Ask the following questions to prompt student discussion:

- What do you know about ratios and proportions?
- Are there any situations you can think of related to these words?
- Is there anything you know of that is proportional?
- What additional words do you associate with these words?

✓ Actively monitor students and provide scaffolding support by having students respond to the following prompts:

- Why do you associate that word to ratios?
- What helped you connect that example to proportions?
- Explain this word to me.
- How does the example use ratios?
Note: The teacher is responsible for encouraging participation from all team members. Some teams may find it difficult to begin. If so, the teacher can ask the following questions to activate prior knowledge:

- What is unit rate?
- How can you represent ratios?
- What do you remember about ratios?

Teams share their Graffiti Wall with the whole class, making connections to other student’s thoughts and ideas and exploring different perspectives.

Allow time for students to discuss ideas they may have shared and/or unique notions presented (SMP.3). Students discuss while the teacher ensures discussions remain math-centered.

✓ Check for understanding as students explain their reasoning. Determine which students have made appropriate connections to prior concepts or skills and which students needed extra support.

Collect and display Graffiti Walls grouped together as one giant wall. This will be displayed for the remainder of the unit and students will have the opportunity to add to it as the unit progresses.

For students who are EL, have disabilities, or perform well below grade-level:

- Use the Graffiti Wall for both visual and written representations. Encourage students to continue to add ideas to the wall throughout the unit.

Extensions for students with high interest or working above grade level:

- Encourage teams with detailed Graffiti Walls to create real-world applications and careers where unit rate would be important to understand.

Activity 1: Unit Rates Video
Display the Unit Rates Video.

Note: This video may be substituted with another on proportionality if you cannot access YouTube.
Have students complete a Think-Pair-Share activity. Facilitate discussion of the information that would help them respond to the question at the end of the video asking students to determine the cost effectiveness of selecting one gas station over another. Once partners share with one another, ask for volunteers to share responses with the whole group. Explain that students can organize the information to solve this problem with a table. Today's lesson will focus on finding unit rate from a table.

**Activity 2: Table as a Tool**

Distribute **Handout 1.2: Table as a Tool**. Display the first table on the board. Instruct students to independently complete the table. Once completed, students compare answers and discuss methods used to solve (SMP.3).

**Note:** The table values are not in any order and will require some perseverance (SMP.1). It is normal for students to use a Guess and Check method. If a student is using this method, suggest exploring by looking for a pattern (SMP.7).

Select students to come to the board and complete the table. Lead a whole group discussion on different methods and reasoning.

- Actively monitor students and provide scaffolding support by asking the following questions:
  - How did you find the missing value?
  - Did you use the same method for every value?
  - Can you show it another way?
  - How does this relate to unit rate?
  - What is the unit rate?
  - Could we create a situation to represent this table?
  - Is the x-axis represented on this table? Explain your answer.
  - Is the y-axis represented on the table? Explain your answer.

Answer any additional questions students have about completing the table. Repeat this process for the remaining tables on the handout.

**Note:** As exposure increases, students will have more questions. Incorporate how each table could represent a real-world scenario.
Each time the class returns to whole group discussion, ask the following questions to help them solidify understanding:

- How are you finding unit rate using the table?
- Is anyone in your team using a different method?
- How could you add more values to your table?
- Is there a limit to the number of values for a unit rate?
- What makes the unit rate a constant rate?
- What would the y-value be if the x-value was zero?
- Is this table proportional?

For students who are EL, have disabilities, or perform well below grade-level:

- Encourage students who are having difficulties finding a pattern to put the x-values (and their y-values) in order from least to greatest to help organize the table.
- Provide students with a multiplication fact sheet or a calculator to assist in the creation of their own table.

Extensions for students with high interest or working above grade level:

- Create more challenging tables using decimals or fractions.
- Ask them to create their own table or values to represent a real-world constant.

✔ Monitor discussions to check for understanding. Determine which students have begun to make connections between the multiplicative factor and the unit rate.

Reflection and Closing:

Refer to the Graffiti Wall and review the lesson with students. Ask the following questions to prompt students:

- What words did we use the most today?
- Can we add anything to our wall?
- What one word will you remember from today?
- Is there anything that we didn’t cover today that we need to look at later?
• What is another way we can define unit rate?
Exit Ticket: Using Handout 1.2: Table as a Tool, have students create their own table with a unit rate of 4.

<table>
<thead>
<tr>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will not receive homework.</td>
</tr>
</tbody>
</table>
Handout 1.1: Graffiti Wall
Handout 1.2: Table as a Tool

Name: ___________________________ Date: _______________

Directions: Fill in the missing values for each table, as directed by the teacher. Describe the strategy you used.

Table 1:

<table>
<thead>
<tr>
<th>x</th>
<th>10</th>
<th>4</th>
<th>9</th>
<th>7</th>
<th>18</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>32</td>
<td>8</td>
<td>16</td>
<td>40</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Strategy Used:

Table 2:

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>5</th>
<th>-1</th>
<th>1</th>
<th>-5</th>
<th>0</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-27</td>
<td></td>
<td>-9</td>
<td>3</td>
<td>15</td>
<td></td>
<td>-24</td>
</tr>
</tbody>
</table>

Strategy Used:

Table 3:

<table>
<thead>
<tr>
<th>x</th>
<th>-9</th>
<th>15</th>
<th>42</th>
<th>18</th>
<th>0</th>
<th>-6</th>
<th>30</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-6</td>
<td>8</td>
<td>12</td>
<td>-4</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Strategy Used:

Create Your Own!

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Answer Key:

**Table 1: \( y = 8x \)**

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>4</th>
<th>1</th>
<th>9</th>
<th>7</th>
<th>2</th>
<th>18</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>80</td>
<td>32</td>
<td>8</td>
<td>72</td>
<td>56</td>
<td>16</td>
<td>144</td>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>

Strategy Used: Strategies will vary. Expect guess and check, rearrangement, and/or using a rule to follow the pattern.

**Table 2: \( y = -3x \)**

<table>
<thead>
<tr>
<th>( x )</th>
<th>9</th>
<th>2</th>
<th>5</th>
<th>3</th>
<th>-1</th>
<th>1</th>
<th>-5</th>
<th>0</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-27</td>
<td>-6</td>
<td>-15</td>
<td>-9</td>
<td>3</td>
<td>-3</td>
<td>15</td>
<td>0</td>
<td>-24</td>
</tr>
</tbody>
</table>

Strategy Used: Strategies will vary. Expect guess and check, rearrangement, and/or using a rule to follow the pattern.

Students may struggle with the negatives if they do not have a strong sense for multiplying integers.

**Table 3: \( y = \frac{2}{3}x \)**

<table>
<thead>
<tr>
<th>( x )</th>
<th>-9</th>
<th>15</th>
<th>12</th>
<th>42</th>
<th>18</th>
<th>0</th>
<th>-6</th>
<th>30</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-6</td>
<td>10</td>
<td>8</td>
<td>28</td>
<td>12</td>
<td>0</td>
<td>-4</td>
<td>20</td>
<td>( \frac{2}{3} )</td>
</tr>
</tbody>
</table>

Strategy Used: Strategies will vary. Expect guess and check, rearrangement, and/or using a rule to follow the pattern. Anticipate students who have difficulties with fractions to struggle with this problem.
Lesson 2: Proportionality in a Table

Focus Standard(s): 7.RP.2a, 7.RP.2b, 7.RP.2c

Additional Standard(s): 7.RP.2d

Standards for Mathematical Practice: SMP.1, SMP.3, SMP.8

Estimated Time: 90 minutes

Resources and Materials:
- Anchor Chart Paper
- Markers (4 colors/team)
- Post-It Notes (1 pad/team)
- Handout 1.2: Table as a Tool (return to students)
- Handout 2.1: Poster Session Task Cards
- Handout 2.2: Exit Ticket
- Handout 2.3: Homework

Lesson Target(s):
- Students will determine whether two quantities are in a proportional relationship by identifying equivalent ratios in a table.
- Students will identify the constant of proportionality using tables and verbal descriptions of proportional relationships.

Guiding Question(s):
- How can I determine whether two quantities are in a proportional relationship?
- How can I identify the constant of proportionality (unit rate) in tables?
- How do I describe the proportional relationship?
Vocabulary

**Academic Vocabulary:**
- Constant of Proportionality
- Proportional
- Rate
- Unit Rate
- Ratio
- Origin
- y-intercept

**Instructional Strategies for Academic Vocabulary:**
- Introduce words with student-friendly definition and pictures
- Model how to use the words in discussion
- Read and discuss the meaning of word in a mathematical context

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type of Text and Interpretation of Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level</td>
</tr>
<tr>
<td>✔</td>
<td>Assessment (Pre-assessment, Formative, Self, or Summative)</td>
</tr>
</tbody>
</table>

**Instructional Plan**

**Understanding Lesson Purpose and Student Outcomes:** Students will work with real-world scenarios to determine proportionality. Students will explore the importance of the origin in a proportional relationship and develop an understanding of linear equations.

**Anticipatory Set/Introduction to the Lesson: Generalizing Patterns**

Return **Handout 1.2: Table as a Tool** to students. Challenge teams to look at the tables from the day before and generalize how they can find the y-value in the tables given any x-value. Have students look for repeated regularity in the tables to formulate an understanding algebraically (SMP.8). Lead students to understand a rule or equation is written in the form of $y=kx$. 
Note: Students may say “x multiplied by unit rate gives you y” instead of formulating an algebraic equation. This basic understanding is acceptable for the first part of the day.

As students work with teams, ask questions to help them solidify their understanding:

- Did you use the same method on every table?
- What do the tables have in common?
- Can we say it in an easier way?
- How can we represent the method using variables?
- Would rearranging the x-values help you find a pattern?

✓ Monitor responses to check for understanding. Determine which students identified the structure of a proportional relationship in the form of \( y=kx \).

Lead a whole group discussion and elaborate on how to write the rules for each table. Instruct students to write the rule for every table in the form \( y=kx \).

For students who are EL, have disabilities, or perform well below grade-level:
- Students share ideas with groups before sharing with the class.

Extensions for students with high interest or working above grade level:
- Have students verify their rule with a variety of rational numbers, e.g., negatives, fractions, decimals.

Activity 1: Determining Proportionality

Display all the tables and their equations on board. Have students discuss with their teams how the tables are the same and how they are different. Verify the understanding that the tables all share a common point, \((0,0)\), the origin. Return the class to whole group and define origin and refer to the Graffiti Wall, reinforcing vocabulary words from the unit.

Facilitate the conversation through questioning. Ask the following questions to prompt students:
• How does the origin affect the unit rate?
• Does each table have a different equation?
• How would you graph the tables?
• How do the equations relate to the table and graph?
• Would putting the x-values in order help you compare the tables?

✓ Monitor discussions to check for understanding. Determine how well students identify the origin as a characteristic of proportionality. While monitoring discussions, encourage students to explain reasons for agreement with others (SMP.3).

Review how to graph from a table. Instruct students to graph Table 1. Reinforce academic vocabulary related to graphing, especially the terms linear, origin, y-intercept.

Note: Make connections between tables, graphs, and equations throughout the unit. Take advantage of any opportunity to build on their knowledge of commonalities between these representations.

For students who are EL, have disabilities, or perform well below grade-level:
• Have students only compare tables to tables at this time.
• Give students a teacher copy of table to ensure they have all correct answers and assist when comparing.
• Provide students with a partially completed and labeled graph or sample.
• Provide students with a multiplication fact sheet to assist in identifying the unit rate of the table.

Extensions for students with high interest or working above grade level:
• Have students graph all the tables on a coordinate plane.
• Encourage students to use more efficient scales on their graphs to reflect the rate of change.
Activity 2: Poster Session
Provide each team with a task from Handout 2.1: Poster Session Task Cards and a piece of anchor chart paper. Assign each team a different task. Give a marker to each student to use on the poster. Instruct students to only use their assigned color during the poster session.

**Note:** This strategy allows teachers to see the contributions of each individual student simply by looking at color distribution.

Tasks require students to copy their original representation and to create either a table or a scenario to accompany it. Have students complete the task with their team and determine whether the scenario shows proportionality or not (SMP.1). Teams hang up their posters upon completion to prepare for Gallery Walk.

For students who are EL, have disabilities, or perform well below grade-level:
- Assign a task card providing a table, so the material is familiar.
- Provide students with a Checklist to make the task more manageable.

Extensions for students with high interest or working above grade level:
- Assign a task card providing a graph or a scenario, which requires students to interpret a situation.

Activity 3: Gallery Walk
**Note:** The Gallery Walk can be done inside the classroom, hallway, or anywhere space provides. Posters need to be spread out so teams can rotate. Each team will need a pack of Post-It Notes.

✓ Provide the following questions to each team for students to provide feedback for other teams:
  - Do you agree with the team’s response to whether the scenario was proportional? Why or why not?
  - If the poster shows a proportional relationship, what is the constant of proportionality?
  - If the poster does not show a proportional relationship, what can be changed in the table or scenario to make it one?
Have teams stand in front of their poster. Instruct teams to rotate to the next poster and provide feedback based on the prompting questions, continuing the process until they have seen the different task cards and critiqued the reasoning of other teams (SPM.3). Teams discuss each poster, and write their thoughts on a Post-It Note to place behind the poster.

Monitor team discussions and ask the following clarifying questions to deepen understanding:

- How do you think this team came to their conclusion?
- Can you show me where they used unit rate?
- Where can you see the origin on the poster?
- How can you relate this scenario to what we have already done in class?
- How do you know it is/isn’t proportional?

After teams visit every poster, have teams return to their poster to review the Post-It Notes others left behind their poster.

**Reflection and Closing:**

Facilitate a discussion about the Gallery Walk. Make notes about students’ understanding and/or misconceptions based on their answers to the following questions:

- Did any comment make your team change your minds?
- Did you see any methods different from your teams?
- Would you have worked the other tasks differently or the same as the other teams did?
- What are some ways to know if a relationship is proportional?
- Did you notice any common traits between the graphs, tables, and equations?

✓ Students complete **Handout 2.2: Exit Ticket.**

✓ Collect these to verify the students’ understanding of the characteristics of proportionality before moving deeper into graphical representations the next day. Review strategies identified on the exit ticket to prepare for clearing up misconceptions the following day.
Homework

Have students complete Handout 2.3: Homework.
Handout 2.1: Poster Session Task Cards

Team 1: A bag of Laffy Taffy contains 2 pieces of strawberry taffy for every 3 green apple taffy.

Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs) comparing different amounts of green apple and strawberry Laffy Taffy a bag could contain, and determine if the relationship between strawberry and green apple Laffy Taffy is proportional. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

Team 2:

<table>
<thead>
<tr>
<th>Parts of Red Paint</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>1/2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parts of White Paint</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

Task: On your poster, copy the table of values comparing the parts of red and white paint, create a real-world scenario to accompany the table, and determine if the relationship between parts of red paint and parts of white paint is proportional. Explain your response using at least two facts about the situation. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

Team 3: Vash plans to order some new games online. The site charges a rate of $25.00 per game for one game, $35 for two, and $40 for three.

Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs), comparing different costs for Vash to order up to ten games, and determine if the relationship the number of games ordered and the total cost is proportional. Explain your response using at least two facts about the situation. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.
Team 4:

<table>
<thead>
<tr>
<th>Miles</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallons</td>
<td>20</td>
<td>19</td>
<td>18</td>
<td>17</td>
<td>16</td>
</tr>
</tbody>
</table>

Task: On your poster, copy the table of values comparing the miles to gallons, create a real-world scenario to accompany the table, and determine if the relationship between miles and gallons is proportional. Explain your response using at least two facts about the situation. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

Team 5: A 28-centimeter-tall candle burns at a rate of 2 centimeters per hour.

Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs), comparing different lengths of time the candle burns and the height of the candle, and determine if the relationship between the candle height and the time is proportional. Explain your response using at least two facts about the situation. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

Team 6: A local restaurant serves 5 hamburgers every 30 minutes they are open.

Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs), comparing different lengths of times the restaurant is open and number of hamburgers served, and determine if the relationship between the candle height and the time is proportional. Explain your response using at least two facts about the situation. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.
Graph Paper for Poster Session
Student Checklist for Poster

☐ I have created a scenario for my task or copied the one I was given.

☐ I have created a table for my task or copied the one I was given.

☐ I have determined whether my task was proportional and written my explanation using complete sentences.

☐ I have identified 2 facts to help me determine why my task was either proportional or nonproportional.

☐ I have graphed my values on the coordinate plane, with numbered and labeled axes.

☐ I organized my information clearly for others to understand.
Team 1: A bag of Laffy Taffy contains 2 pieces of strawberry taffy for every 3 green apple taffy. Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs), comparing different amounts of green apple and strawberry Laffy Taffy a bag could contain, and determine if the relationship between strawberry and green apple Laffy Taffy is proportional. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

<table>
<thead>
<tr>
<th>Green Apple</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberry</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

This is an example of a proportional relationship. The table contains the origin, (0, 0). It has a constant growth rate, because for every 3 green apple pieces it has 2 strawberry pieces. The strawberry taffy amount is always \( \frac{2}{3} \) of the green apple amount.

Team 2:

<table>
<thead>
<tr>
<th>Parts of Red Paint</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>( \frac{1}{2} )</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parts of White Paint</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

Task: On your poster, copy the table of values comparing the parts of red and white paint, create a real-world scenario to accompany the table, and determine if the relationship between parts of red paint and parts of white paint is proportional. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

Scenarios will vary. This is a proportional relationship. If the painter added 0 parts of red paint, then he/she would not add any white, which means the table would contain the origin, (0, 0). The white paint is always four times the amount of the red paint, meaning it has a unit rate/constant of proportionality. The equation would not contain any addition or subtraction. It would just be \( y = 4x \).

Team 3: Vash plans to order some new games online. The site charges a rate of $25.00 per game for one game, $35 for two, and $40 for three.
Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs), comparing different costs for Vash to order up to ten games, and determine if the relationship the number of games ordered and the total cost is proportional. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

<table>
<thead>
<tr>
<th>Number of Games</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>0</td>
<td>$25</td>
<td>$35</td>
<td>$40</td>
</tr>
</tbody>
</table>

This is not an example of a proportional relationship. The situation does not have a constant rate of change.

Team 4:

<table>
<thead>
<tr>
<th>Miles</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallons</td>
<td>20</td>
<td>19</td>
<td>18</td>
<td>17</td>
<td>16</td>
</tr>
</tbody>
</table>

Task: On your poster, copy the table of values comparing the miles to gallons, create a real-world scenario to accompany the table, and determine if the relationship between miles and gallons is proportional. Graph your values and number and label your graph’s axes. Explain your response using at least two facts about the situation.

This is not proportional. Even though it seems to have a constant rate of change (30 mpg), the graph does not begin at the origin (0, 0). The gallons and the miles both need to be at (0, 0) for this situation to work. This graph is linear, but not proportional.

Team 5: A 28-centimeter-tall candle burns at a rate of 2 centimeters per hour.

Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs), comparing different lengths of time the candle burns and the height of the candle, and determine if the relationship
between the candle height and the time is proportional. Graph your values and number and label your graph’s axes. Explain your reasoning using at least two facts about the situation.

<table>
<thead>
<tr>
<th>Number of Hours Burning</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of the Candle (cm)</td>
<td>28</td>
<td>26</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>

This is not a proportional relationship. The candle is burning at a constant rate of 2 cm per hour; however, proportionality requires it go through the origin (0, 0) and this situation does not. This is linear, but not proportional.

Team 6: A local restaurant serves 5 hamburgers every 30 minutes they are open.

Task: On your poster, write your team’s scenario, create a table of values (with at least 4 sets of ordered pairs). comparing different lengths of times the restaurant is open and number of hamburgers served, and determine if the relationship between the candle height and the time is proportional. Graph your values and number and label your graph’s axes. Explain your reasoning using at least two facts about the situation.

<table>
<thead>
<tr>
<th>Time Opened (minutes)</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Hamburgers Served</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

This is a proportional relationship. It begins at (0, 0). It has a constant growth rate of 5 hamburgers/30 minutes.
Handout 2.2: Exit Ticket

Name: ___________________________ Date: __________

List three ways to determine proportionality:

________________________________________

________________________________________

________________________________________

Explain one strategy to find the constant of proportionality:

________________________________________
Handout 2.3: Homework

Name: __________________________________________

Part 1

1. Tim grows sunflowers from seeds, but not all of his seeds start to grow. He has found that for every 100 seeds he sows, only about 75 start to grow.

   a. Tim sows 20 sunflower seeds. How many should he expect to grow? Explain your reasoning.

   b. Tim sows 24 seeds in a box. Each mark on the box below shows the position of a seed.

       ![Diagram of seeds]

       Guess which of the seeds start to grow. Draw circles around the seeds that do not start to grow. (Note: There is more than one correct way to show your answer to this question.)

       Explain your reasoning.

Part 2

2. a. Fill in the table showing the number of seeds planted compared to the number of seeds that start to grow. Provide an equation for the table. _______________

<table>
<thead>
<tr>
<th>Number of Seeds Planted (x)</th>
<th>0</th>
<th>20</th>
<th>60</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Seeds That Grow (y)</td>
<td>30</td>
<td>75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MS Exemplar Unit ● Mathematics

Grade 7 ● Edition 2
Part 1

1. Tim grows sunflowers from seeds, but not all of his seeds start to grow. He has found that for every 100 seeds he sows, only about 75 start to grow.

   a. Tim sows 20 sunflower seeds. How many should he expect to grow? Explain your reasoning.
      Tim should expect 15 sunflowers to grow.
      Students may set up a proportion to solve for the unknown, or they may find the unit rate (constant of proportionality) is \( \frac{3}{4} \) and multiply it to 20, the total number of seeds.

   b. Tim sows 24 seeds in a box. Each mark on the box below shows the position of a seed.

   ![Diagram of seeds]

      Guess which of the seeds start to grow. Draw circles around the seeds that do not start to grow. (Note: There is more than one correct way to show your answer to this question.) Explain your reasoning.
      Circles and reasoning may vary, but it is important for students to have 6 marks circled to indicate the \( \frac{1}{4} \) of the seeds that will not grow based on the constant of proportionality.

Part 2

2. a. Fill in the table showing the number of seeds planted compared to the number of seeds that start to grow. \( y = \frac{3}{4}x \)

<table>
<thead>
<tr>
<th>Number of Seeds Planted ((x))</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Seeds That Grow ((y))</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
<td>75</td>
<td>90</td>
</tr>
</tbody>
</table>

Although it makes it easier to follow the pattern, students do not have to provide the pairs (80, 60) and (120, 90). They may provide any pair that satisfies the rule.
Lesson 3: Comparing Tables and Graphs

Focus Standard(s): 7.RP.2b, 7.RP.2c, 7.RP.2d

Additional Standard(s): 7.RP.1

Standards for Mathematical Practice: SMP.1, SMP.3, SMP.5, SMP.8

Estimated Time: 90 minutes

Resources and Materials:
- Ruler
- Graph Paper
- Handout 3.1: Characteristics of Proportionality
- Handout 3.2: Comparing Tables and Graphs
- Handout 3.3: The Importance of (1, y)
- Desmos Graphing Calculator: www.desmos.com/calculator

Lesson Target(s):
- Students will discover what makes two quantities proportional.
- Students will recognize proportional relationships on tables and graphs.

Guiding Question(s):
- How can you determine if two quantities are proportional?
- How can you recognize a proportional relationship on a table and a graph?

Vocabulary

<table>
<thead>
<tr>
<th>Academic Vocabulary:</th>
<th>Instructional Strategies for Academic Vocabulary:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Unit Rate</td>
<td>□ Introduce words with student-friendly definitions and pictures</td>
</tr>
<tr>
<td>• Constant of Proportionality</td>
<td></td>
</tr>
<tr>
<td>• Proportional</td>
<td>□ Model how to use the words in discussion</td>
</tr>
</tbody>
</table>
Understanding Lesson Purpose and Student Outcomes: Students will use what they have previously learned to compare different representations of proportional relationships. Students will identify traits of proportionality in tables and graphs.

Anticipatory Set/Introduction to the Lesson: Real-World Proportions
Instruct students to create a real-world scenario they believe to be proportional. Select students to share examples with the class and discuss whether they agree with examples.

Facilitate group discussion using the following prompting questions:
- Do you agree that this situation represents a proportional relationship?
- What indicates that this is a (non)proportional relationship?
- What is the rule for your relationship?
- Does your relationship include the origin?
- Give students these ordered pairs and ask if they can find out if they represent a proportional relationship. Discuss.

\[(\frac{1}{4}, \frac{1}{2}) \quad (\frac{1}{2}, 1) \quad (\frac{3}{4}, \frac{3}{2})\]

Activity 1: Characteristics of Proportional Graphs
Distribute Handout 3.1: Characteristics of Proportionality. Have students independently create a table of values for each situation.

Note: Give students additional support as needed when they encounter scenarios with y-intercepts that are not the origin. Ask the following guiding questions to assist in making connections between the characteristics of proportionality and the representations:
• Is the y-intercept the origin?
• If the x-value is zero, does that mean the y-value will also be zero?
• When a baby is born (0 days old), is the weight of the baby zero?
• If you spend zero hours working, what will you accomplish?
• How did you choose to set up the table?
• Does your table contain a unit rate?
• Is the relationship linear?

Lead a whole group discussion. Select students to share and justify responses for each scenario analyzed (SMP.3).

Explain that a table is only one method of representation. Ask students to brainstorm another representation they can create from a table. Verify student understanding of creating graphs from a table. Instruct students to create graphs for the three tables they used.

**Note:** Model how students should organize the coordinate plane with title, labels, and scaling (SMP.5).

Facilitate whole group instruction comparing tables and graphs. Ask students to identify which graphs are proportional. Have students list what characteristics the proportional graphs have that differ from the nonproportional graphs. Verify that students identify that part (b) does not begin at the origin and part (c) does not begin at the origin and is curved/non-linear.

Invite students to add new ideas to the class Graffiti Wall.

**For students who are EL, have disabilities, or perform well below grade-level:**

• Students may need assistance identifying the independent and dependent variables in each situation. Provide these students with notes on determining independent and dependent variables, with real-world visual examples.

**Activity 2: Comparing Tables and Graphs**

Distribute **Handout 3.2: Comparing Tables and Graphs**. Have students work with their team to answer the situations provided. Tell teams to read the discussion questions and be ready to explain their reasoning.

✓ Actively monitor team progress while students determine if the situation is proportional, create a table from the situation, compare a table to a graph to determine who gets paid more, and identify unit rates (SMP.1).
Note: In this problem, students deal with money. They need a strong sense of rounding to get the precise amounts. They also need to be able to find a unit rate using multiple representations of proportionality.

For students who are EL, have disabilities, or perform well below grade-level:
- Allow students to use the Desmos Graphing Calculator to create the graphs if needed.
- Provide students with a multiplication fact sheet to assist in finding unit rates.

Extensions for students with high interest or working above grade level:
- Have students create a situation for a third trainer who makes more than Regina, but less than Hai. Instruct them to represent the third trainer using an equation, table, and graph.

Activity 3: (1, y) Turn and Talk
Provide Handout 3.3: The Importance of (1, y) for students to complete independently. After students complete independently, instruct students to Turn and Talk about the discussion question on the handout. Actively monitor discussions and listen for an understanding of (1, y) identifying the unit rate.

Facilitate whole group discussion on (1, y) using the following prompting questions:
- What is (1, y) referring to?
- Can you show me the unit rate on the graph?
- Can you use other points to determine the unit rate? How?
- How did you use the information in the scenario to identify the unit rate?

Activity 4: Connecting the Representations
Direct students to refer to the equations they wrote for tables in the previous lesson. Ask students to identify what the equation has in common with the unit rate (constant of proportionality). Validate student understanding that the value of the unit rate is the same value being multiplied to by the x-value (SMP.8). Reinforce the notion of constant of proportionality and unit rate are the same.
value and how equations for proportional relationships are written in the form $y = kx$, where $k$ represents the constant of proportionality. Ask students to reflect and share on how the equation $y = kx$ satisfies the characteristics of proportionality.

Display the problem below for students to complete:
T: “Alexia wrote the equation $d = \frac{5}{6}w$ to represent the amount of a bottle of water (w) she consumes in any number of days (d).”

Allow students time to write down important information and ask clarifying questions as needed.  
T: “How much water does Alexia drink in 3 weeks?”
Copy the following table on the board:

<table>
<thead>
<tr>
<th>Alexia’s Brother</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (days)</td>
</tr>
<tr>
<td>Bottle of Water (#)</td>
</tr>
</tbody>
</table>

T: “Alexia’s brother kept track of his water consumed in the table. Who drinks more water, Alexia or her brother?”

✓ Monitor students and check for understanding using the following prompting questions:
- How do you know Alexia’s unit rate?
- How did you calculate Alexia’s brother’s unit rate?
- Whose rate is easier to determine? Explain your reasoning.
- Can you write an equation for Alexia’s brother?
- Can you make a table for Alexia?
- How would you graph these?
- What is the $y$-intercept of each and what does it tell you about the situation?
- Are these relationships proportional? How do you know?

**Reflection and Closing:**
Select volunteers to share their answers from Activity 4 to close the day’s lesson. Facilitate the discussion to center around comparing two different representations of proportional relationships using the following prompting questions:
- How were you able to compare a table and an equation?
- How would you compare a graph to a table or equation?
- What can you conclude about proportional relationships?
- How are the different representations compared?

**Homework**

Students will create 3 proportional relationships. One must be a table, one a graph, and one an equation.
Handout 3.1: Characteristics of Proportionality

Name: ________________________________ Date: _____________

Independent Work:

1. Directions: Make a table to represent each of the situations below.
   a. Angelina is washing dishes at work. She can wash 15 cups in 10 minutes. How many cups can she clean in different amounts of time?

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cups Washed</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Miles’ cat, Cinnamon, weighed 12 ounces at birth. Cinnamon doubled her weight in 6 days. Assuming her growth is constant, how much will Cinnamon weigh at various times in the first year?

<table>
<thead>
<tr>
<th>Time (Days Since Birth)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (ounces)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c. Suleiman’s bank advertises that it will double your money invested every year. If Suleiman invests the $20 his father gave him for his birthday, how much money will he have in various years?

   |   |   |   |   |

   |   |   |   |   |

Group Discussion Questions:

2. Which of the situations above describe proportional relationships?

3. How do you know by looking at the table?

4. Do you think looking at a graph instead of a table would make this easier to determine? Why or why not?
Answer Key

1. Directions: Make a table to represent each of the situations below.
   a. Angelina is washing dishes at work. She can wash 15 cups in 10 minutes. How many cups can she clean in different amounts of time?

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cups Washed</td>
<td>0</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

   b. Miles’ cat, Cinnamon, weighed 12 ounces at birth. Cinnamon doubled her weight in 6 days. Assuming her growth is constant, how much will Cinnamon weigh at various times in the first year?

<table>
<thead>
<tr>
<th>Time (Days Since Birth)</th>
<th>0</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (ounces)</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

   Expect students to struggle with (0, 12). Ask provoking questions about something being born and its weight at the time.

   c. Suleiman’s bank advertises that it will double your money invested every year. If Suleiman invests the $20 his father gave him for his birthday, how much money will he have in various years?

<table>
<thead>
<tr>
<th>Time (Years)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>160</td>
<td>320</td>
</tr>
</tbody>
</table>

   Students may struggle with (0, 20). Ask students if Suleiman does not invest his money, how much will he have? His money does not disappear if he doesn’t invest it with his bank. Also, look for students who add $20 every time instead of doubling the previous amount.

Group Discussion Questions:

2. Which of the situations above describe proportional relationships?
   “A” is the only proportional relationship above. Angelina washes at a rate of 15 cups/10 mins (unit rate of 1.5 cups/minute)

3. How do you know by looking at the table?
   “B” cannot be proportional because the cat’s starting weight is (0, 12), which means it will not pass through the origin. “C” also does not pass through the origin, nor does it have a constant of proportionality, since every 1 year is growing by a different amount.

4. Do you think looking at a graph instead of a table would make this easier to determine? Why or why not?
   Answers will vary. Use responses to guide teacher questioning in Activity #2.
Handout 3.2: Comparing Tables to Graphs

Name: ________________________________ Date: __________

1. Hai regularly trains a client at his gym. Last month, Hai’s client paid him $125 for 8 sessions of training.
   a. Is this situation most likely a proportional relationship? How do you know?

   b. Make a table that shows how much Hai earns for various numbers of training sessions.

   c. Regina brags that she is paid more than Hai to train her client. Regina is paid according to the graph below. Who is paid more? How do you know?

   Regina’s Personal Training

   ![Graph showing cost vs. number of training sessions]

   d. At what unit rate do Hai and Regina each get paid? How do you know? Remember that unit rate compares the change in one quantity to a one-unit change in another quantity, so in this case you would compare dollars to one hour.
Answer Key

1. Hai regularly trains a client at his gym. Last month, Hai’s client paid him $125 for 8 sessions of training.
   a. Is this situation most likely a proportional relationship? How do you know?
      Yes, Hai would get paid $0 for 0 training sessions and he’s getting paid the same amount for every training session.
   b. Make a table that shows how much Hai earns for various numbers of training sessions.

      | Number of Training Sessions | 0   | 4   | 8   | 12  | 16  |
      |----------------------------|-----|-----|-----|-----|-----|
      | Cost for Client ($)        | 0   | 62.50 | 125.00 | 187.50 | 250.00 |

      Students may choose whatever number of sessions they want with, as long as they follow the rule $y=15.625x$. Because this is money, students need to round to the hundredths place on their table.

   c. Regina brags that she is paid more than Hai to train her client. Regina is paid according to the graph below. Who is paid more? How do you know?

      Hai is getting paid more. Hai is making $15.63/training session and Regina is making $14.00/training session. I know this because … student answers will vary.

   d. At what unit rate do Hai and Regina each get paid? How do you know? Remember that unit rate compares the change in one quantity to a one-unit change in another quantity, so in this case you would compare dollars to one hour.
      Hai: $15.63/session; Regina: $14.00/session. Reasons will vary.
Handout 3.3: The Importance of (1, y)

Name: ___________________________ Date: ___________

Nadia’s pet dog eats so much that Nadia buys dog food in bulk. Twenty-five pounds of dog food costs $15.75 and fifty pounds of food costs $31.50. $13.75 and thirty-five pounds costs $19.25.

Individual Task:

1. **On the back, make**  
   Make a graph of the cost of dog food for various sizes of bags. Be sure your graph includes a title, labels, and an appropriate scale.

2. **Calculate the value of y at the point (1, y).** Why would Nadia want to know the value of y?

Pair-Share Discussion Topics:

Compare the graphs you and your partner made for the situation above. Did you use the same scale or different scales?

Compare your value of y. Did you get the same value? Explain to your partner what method you used to find the value of y. What does that value tell Nadia?

Note: I made this change to make the rate more realistic (not a fraction of a cent) and the range more manageable (0-35 instead of 0-50)
Nadia’s pet dog eats so much that Nadia has to buy dog food in bulk. Twenty-five pounds of dog food costs $15.75 and fifty pounds of food costs $31.50. $13.75 and thirty-five pounds costs $19.25.

Individual Task:

1. Make a graph of the cost of dog food for various sizes of bags. Be sure your graph includes a title, labels, and an appropriate scale.

   Graphs should be linear following the rule \( y = 0.63x \) or \( y = 0.55x \).

2. Calculate the value of \( y \) at the point \((1, y)\). Why would Nadia want to know the value of \( y \)?

   The value of \( y \) is 0.63. Nadia would want to know this because it tells her that for every 1 pound of dog food she buys she will pay $0.63. The value of \( y \) is 0.55. Nadia would want to know this because it tells her that for every 1 pound of dog food she buys she will pay $0.55.
Lesson 4: Multiple Representations

Focus Standard(s): 7.RP.2a, 7.RP.2b, 7.RP.2c
Additional Standard(s): 7.RP.1
Standards for Mathematical Practice: SMP.1, SMP.3, SMP.4, SMP.6, SMP.7
Estimated Time: 50 minutes

Resources and Materials:
- Color Tiles
- Handout 4.1: What’s the Proportion?
- Handout 4.2: Units of Proportionality
- Handout 4.3: Reynaldo’s Trip
- A Family Guide for Student Success: [https://mdek12.org/OAE/OEER/FamilyGuidesEnglish](https://mdek12.org/OAE/OEER/FamilyGuidesEnglish)

Lesson Target(s):
- Students will create a proportional scenario and prove proportionality using a variety of representations.
- Students will compare constants of proportionality in multiple representations.

Guiding Question(s):
- How can you determine whether two quantities are in a proportional relationship by testing for equivalent ratios in a table?
- How can you identify the constant of proportionality in tables and verbal descriptions of proportional relationships?

Vocabulary

Academic Vocabulary:
- Constant of Proportionality
- Proportional
- Ratio
- Unit Rate

Instructional Strategies for Academic Vocabulary:
- Model how to use the words in discussion
- Create pictures/symbols to represent words
- Write/discuss using the words
Symbol | Type of Text and Interpretation of Symbol
---|---
| Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level

| ✔ | Assessment (Pre-assessment, Formative, Self, or Summative)

**Instructional Plan**

**Understanding Lesson Purpose and Student Outcomes:** Students will create a situation, table, graph, and equation when provided with a unit of measurement. Students will compare unit rates in a real-world context of fuel efficiency.

**Anticipatory Set/Introduction to the Lesson: Tile Patterns**

Distribute **Handout 4.1: What’s the Proportion?** and color tiles. Provide time for students to represent the tile patterns using the color tiles if needed to make sense between the pattern and the unit rate (SMP.4).

✓ Actively monitor students and ask the following prompting questions to deepen understanding of a constant rate (SMP.7):
  - How many tiles does your Figure 0 contain? Why is this important to know?
  - Can you build the pattern with your color tiles?
  - How many new tiles do you need to build the next figure?
  - Are you always adding the same number of tiles to build the next figure?

For **students who are EL, have disabilities, or perform well below grade-level:**

- Students can display information in the representation they find easiest to work with (table or graph).
Activity 1: Units of Proportionality

Note: Cut Handout 4.2: Units of Proportionality into strips before the lesson. Have one Team Checklist sheet per team.

Assign a strip containing units to each team. Instruct students to work as a team to create a table, graph, equation, and real-world scenario using the units provided. Reinforce important elements of graphs (title, labels, and scale) and the proportional equation being written in $y = kx$ form. Encourage students to model the unit rate in each representation (SMP.4).

- Actively monitor group progress and engage all team members in conversation to gauge their understanding of proportionality with the following questions:
  - What are the variables in your problem? What does the $x$-variable in your problem represent? What does the $y$-variable in your problem represent?
  - How can you see the $y$-intercept in your representations?
  - How did you determine the unit rate?
  - How did you select a scale for your graph?
  - What is your unit rate and how is it represented?

Return to whole group and facilitate team sharing. Allow teams to give feedback (SMP.3).

For students who are EL, have disabilities, or perform well below grade-level:
- Provide teams with units they have familiarity with in a real-world context (miles per gallon).
- Scaffold by having teams first discuss where they’ve experienced the units before, then move into putting that experience into words for the scenario.

Extensions for students with high interest or working above grade level:
- Encourage students to provide rational numbers as their constant of proportionality.
- Allow time for students to research realistic figures to use with their scenario.

Activity 2: Reynaldo’s trip

Distribute Handout 4.3: Reynaldo’s Trip. Instruct students to independently complete the task (SMP.6).
Note: Do not split the task in two days. If the students do not have ample time to complete on day one, save the activity for the next day.

Reflection and Closing:
T: “How well can you use multiple representations to determine if a relationship is proportional?” Effective learners assess their own learning to help them determine their level of understanding.

✓ Have students rate on a scale of 1-5 (1 meaning “not at all” and 5 meaning “very well” on a sheet of paper or note card. Students must reveal this number to you on their way out the class.

Note: This student self-assessment allows teacher to see which students are aware of their weaknesses and are owning their need for growth. If you notice students rating their ability higher than reality, make note of those students to provide extra assistance. Also make note of students who rate themselves on a 1-3 level. Determine a plan of action to help them grow.

Homework
Students will complete the “Help at Home” activity on the top of page 18 of the 7th Grade Family Guide for Student Success.
Handout 4.1: What’s the Proportion?

Name: ________________________________ Date: ______________

1. Describe how the pattern of blue tiles change from one pattern to the next.

2. For the patterns shown, is the number of blue tiles proportional to the pattern number? Explain your reasoning.

3. Describe how the pattern of orange tiles changes from one pattern to the next.

4. For the patterns shown, is the number of orange tiles proportional to the pattern number? Explain your reasoning.

Answer Key
1. Describe how the pattern of blue tiles change from one pattern to the next.
   The number of tiles is increasing by 3 each time.

2. For the patterns shown, is the number of blue tiles proportional to the pattern number? Explain your reasoning.
   Yes, it has a constant growth rate (constant of proportionality) of 3 and its initial value (Figure 0) has zero tiles, meaning it crosses through the origin. The equation would be \( y = 3x \).

3. Describe how the pattern of orange tiles changes from one pattern to the next.
   The pattern has a growth rate of 2 tiles.

4. For the patterns shown, is the number of orange tiles proportional to the pattern number? Explain your reasoning. No. Even though the pattern has a constant growth rate of two, it would not pass through the origin, since the y-intercept (Figure 0) would be 1.
Handout 4.2: Units of Proportionality

Directions: Cut the following units of measurement out along the dotted lines and fold in half. Allow teams to randomly select one from the list below.

- miles per hour
- miles per gallon
- height per year
  In first 10 years
- weight per year
  In first 10 years
- degrees per month
- steps walked per hour
- Calories burned per minute of workout
Team Checklist

- We have created a table for our unit.

- We have created a graph for our unit on the back of this page with a title, labels, and scale.

- We have created an equation for our unit using the form \( y = kx \). _________________

- We have created a real-world scenario for our unit.

We can answer the following questions:

- What does the \( x \)-variable in your problem represent? What does the \( y \)-variable in your problem represent?
- How can you see the \( y \)-intercept in your representations?
- How did you determine the unit rate?
- How did you select a scale for your graph?
- What is your unit rate and how is it represented?
Handout 4.3: Reynaldo’s Trip

Name: ___________________________ Date: ____________

Reynaldo’s Trip

Reynaldo and his friend Katie are planning to drive from New York to San Francisco in his car. Reynaldo started to complete the table below showing how far in miles he can travel for each gallon of gas he uses in his car. After seeing his table, Katie suggested they take her car instead because she believes her car gets more miles per gallon. Use the information to determine which car they should take on their trip.

Reynaldo’s Car

1. Complete the table representing Reynaldo’s Car.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>56</td>
<td>168</td>
<td>224</td>
<td></td>
</tr>
</tbody>
</table>

2. Based on the table, how many miles per gallon does Reynaldo’s car get? How did you use the table to solve?

3. Is the relationship on the table proportional? Explain how you know.
4. Katie’s car is represented using the graph below. How many miles per gallon does Katie’s car get?

5. Is the relationship represented on the graph proportional? Explain how you know.

6. Using the information gathered, whose car should Reynaldo and Katie take to San Francisco? Explain your reasoning using an equation for both cars.
Reynaldo and his friend Katie are planning to drive from New York to San Francisco in his car. Reynaldo started to complete the table below showing how far in miles he can travel for each gallon of gas he uses in his car. After seeing his table, Katie suggested they take her car instead because she believes her car gets more miles per gallon. Use the information to determine which car they should take on their trip.

**Reynaldo’s Car**

<table>
<thead>
<tr>
<th>Gallons</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>56</td>
<td>112</td>
<td>168</td>
<td>224</td>
<td>280</td>
<td>336</td>
</tr>
</tbody>
</table>

*Answers vary for last field, but must be proportional to the relationship demonstrated in the table.*

Use the information in Reynaldo’s table to answer the questions below.

2. Based on the table, how many miles per gallon does Reynaldo’s car get? How did you use the table to solve?

   Reynaldo’s car gets 28 miles per gallon. I was able to divide miles by gallons and all the pairs simplified to 28.

3. Is the relationship on the table proportional? Explain how you know.

   Yes. Reynaldo’s car has a constant of proportionality of 28, which means it is a linear relationship. If he drives 0 miles he uses 0 gallons, so that means it crosses through the origin. This makes it proportional.
4. Katie’s car is represented using the graph below. How many miles per gallon does Katie’s car get?

![Graph showing miles per gallon](https://via.placeholder.com/150)

Katie’s car gets 25 miles per gallon.

7. Is the relationship represented on the graph proportional? Explain how you know.

Yes. It has a constant of proportionality of 25, it is linear and it begins at the origin.

8. Using the information gathered, whose car should Reynaldo and Katie take to San Francisco? Explain your reasoning.

Reynaldo’s car gets 28 miles to the gallon or $y = 28x$, while Katie’s car only gets 25 miles to the gallon or $y = 25x$, so they should take Reynaldo’s car.
Lesson 5: Solving Proportional Relationships

Focus Standard(s): 7.RP.2b, 7.RP.2c
Additional Standard(s): 7.RP.1

Standards for Mathematical Practice: SMP.1, SMP.2, SMP.4, SMP.6, SMP.8

Estimated Time: 90 minutes

Resources and Materials:
- Colored Paper
- Markers
- Scissors
- Glue
- Handout 5.1: Bellwork
- Handout 5.2: Teacher Notes on Multiple Strategies
- Handout 5.3: Using Equivalent Ratios
- Handout 5.4: Find Someone Who
- Handout 5.5: Pocket Book Instructions
- Handout 5.6: Card Sort
- Handout 5.7: Frayer Model

Lesson Target(s):
- Students will solve problems using equivalent ratios.
- Students will solve proportions using multiple strategies, including linear graphs.

Guiding Question(s):
- How can you determine whether two quantities are in a proportional relationship by testing for equivalent ratios in a table?
- How can you identify the constant of proportionality in tables and verbal descriptions of proportional relationships?
### Vocabulary

**Academic Vocabulary:**
- Equivalent
- Ratio
- Proportional

**Instructional Strategies for Academic Vocabulary:**
- Introduce words with student-friendly definition and pictures
- Model how to use the words in discussion
- Read and discuss the meaning of words in a mathematical context
- Create pictures/symbols to represent words
- Write/discuss using the words

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Type of Text and Interpretation of Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level</td>
</tr>
<tr>
<td>✓</td>
<td>Assessment (Pre-assessment, Formative, Self, or Summative)</td>
</tr>
</tbody>
</table>

### Instructional Plan

**Understanding Lesson Purpose and Student Outcomes:** Students will learn multiple strategies for solving proportional relationships.

**Anticipatory Set/Introduction to the Lesson: Ratios of Users**

Distribute **Handout 5.1: Bellwork.**

- ✓ Actively monitor students working independently to understand how a country with more people with cellphones has a lower ratio of cellphone users (SMP.1).

- ✓ Actively monitor and listen to team discussions comparing the number of cellphone users to the total population. Verifying an understanding of unit rate and how it relates to the population by asking the following prompting questions (SMP.4):
• How do the scenarios compare?
• How does the number of cellphones affect the ratios?
• How would a graph help represent this situation?
• How does the total population change your ratio?
• Which country is larger?

For students who are EL, have disabilities, or perform well below grade-level:
• Ask guiding questions about what is being compared to create the ratio.

Extensions for students with high interest or working above grade level:
• Students may look up populations and number of cellphones to prove theorems.

Activity 1: Using Equivalent Ratios

Note: Review Handout 5.2: Teacher Notes on Multiple Strategies and become familiar with a variety of strategies students can use to solve proportional relationships. Facilitate whole group discussion on identifying proportional relationships in tables, graphs, and equations. Reinforce the characteristics of proportionality for students. Explain to students that when working with proportional relationships, there will be times when they will need to solve for a missing variable in one of the ratios. Provide students multiple methods for solving proportions, since some problems will include fractional answers. Encourage students to write solutions in fractional form to obtain the most precise answer, as well as use units where necessary (SMP.6).

T: Write this table on the board. Ask students to find the missing value if the values are proportional.

<table>
<thead>
<tr>
<th>Days</th>
<th>Kai Miles Run</th>
<th>Ran</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Then write: \( \frac{2}{5} = \frac{x}{15} \) and discuss the relationship to the table.
Allow students quiet think time to come up with ways to determine the value of x. Expect students with strong backgrounds solving equations to multiply by 15, while others may see that 15 is 3 times larger than 5, which means they need to find a number 3 times
larger than 2 for x (SMP.2 and SMP.8). Select students to share strategies with the class. Discuss any unique ideas and what mathematics supported their reasoning. Present students with additional strategies found in Handout 5.2: Teacher Notes on Multiple Strategies. These strategies include the Giant One, Undoing Division, Clearing Fractions, and Cross-Multiplication.

Pair students and distribute Handout 5.3: Using Equivalent Ratios to each pair. Upon completion, pairs will share with others and discuss which method is preferred.

✓ Actively monitor pairs and use the following prompts to engage all students:
  • Explain why you used that method.
  • Does the size of your numbers affect how you solve the ratio?
  • What is the equation for the table?
  • What is the y-intercept?
  • Does the unit rate change?

✓ Allow students time to reflect on which method they preferred when solving proportional equations. Have students share this self-assessment with their reasoning. Correct any misconceptions and validate any understandings. Determine if more support is needed for individuals or the whole class.

Activity 2: Find Someone Who
Distribute Handout 5.4: Find Someone Who. Allow students 10-15 minutes to fill in the three boxes of the table under the “My Way” column. Then, after 15 minutes, instruct students to stand up and find 3 people in the room who can solve each problem using a different method. Explain to students they cannot go to the same student more than once, so it is important for students to be selective of who they ask to work each problem. Instruct students to turn in sheet to teacher once all boxes are complete.

Note: This activity can be turned into a game, with rewards for students who complete the table first.

Collect all sheets and present one problem at a time, allowing students to present 4 different methods for solving. This will ensure the teacher exposes the class to multiple methods.
For students who are EL, have disabilities, or perform well below grade-level:
- Because the sheet allows the student to choose the first method, he/she may select what they find the easiest. As other students sign their paper using another method, they will be exposed to multiple strategies.
- Allow students to work in pairs to receive more support.

Extensions for students with high interest or working above grade level:
- Students consider methods outside of their comfort zone at times as they look for a different strategy. Encourage more advanced students to be Alternative Method #3, so they must dig deeper to find an unused method.

Activity 4: Pocket Book
✓ Provide students with one piece of colored paper, glue and Handout 5.5: Pocket Book Instructions. Have students assemble the pocket book and title the book as, “Proportional Relationships.” On the inside, have students label the left pocket as “Proportional” and the right pocket as “Non-Proportional.”

Distribute Handout 5.6: Card Sort and scissors. Instruct students to cut cards and sort them into two piles, “Proportional” and “Not Proportional.” Before determining which pocket a card goes in, have students write their reasoning on the back of the card.

For students who are EL, have disabilities, or perform well below grade-level:
- Allow students to work in pairs.

Extensions for students with high interest or working above grade level:
- Have students use index cards to create their own examples of proportional and not proportional relationships.
Note: Additional time may be needed for students to practice the skills addressed this lesson. Use the formative assessments provided to determine which skills may need reinforcement before beginning Lesson 6. Continue to expose students to problems similar to those in Lesson 5 to allow students to develop fluency in solving proportions.

Reflection and Closing:
Encourage reflection on new material through journal writing. Supply students with a prompt explaining their experiences over the course of the unit as they have learned more about proportional relationships.

Allow students time to add new material to the Graffiti Wall.

For students who are EL, have disabilities, or perform well below grade-level:
- Responses can be written in a graphic organizer instead of paragraph form.

Homework

✓ Students will complete Handout 5.7: Frayer Model on Proportionality.

For students who are EL, have disabilities, or perform well below grade-level:
- Have samples of examples and non-examples for students to organize instead of create.
Handout 5.1: Bellwork

Name: ________________________________       Date: ____________

Word Problem:

If India has more cellphones than Italy, how is it possible that Italy has the higher ratio of cellphone owners?

Describe how this is possible:
**Word Problem:**

If India has more cellphones than Italy, how is it possible that Italy has the higher ratio of cellphone owners?

Describe how this is possible:

If the population of India is much higher than that of Italy, a lower ratio could still equal a higher number of cellphones.

Ex. India Population = 1.252 billion and Italy Population = 59.83 million

Because the population is so much larger, multiplying a unit rate that is a proper fraction will keep mean there are more cellphones in India, even though Italians are more likely to own a phone.
Handout 5.2: Teacher Notes on Multiple Strategies

**Giant One Method**

The Giant One method provides students with a visual of using the identity property of multiplication to create equivalent fractions. Students often do this without giving it a name.

Ex. \( \frac{x}{16} = \frac{9}{2} \)

\[
\begin{align*}
\frac{x}{16} &= \frac{9}{2} \cdot 1 \\
\frac{x}{16} &= \frac{9 \cdot 8}{2 \cdot 8} \\
\frac{x}{16} &= \frac{72}{16}
\end{align*}
\]

Since both denominators are now the same, that means the numerators are also equivalent, therefore,

\( x = 72 \).

**Note:** Students may use a Giant One on both sides of the proportion if needed.

**Undoing Division Method**

The Undoing Division method is used to remove the denominator on under the variable. This is done by multiplying both sides of the proportion by the denominator, just as students would do to solve a one-step equation.

Ex. \( \frac{5}{7} = \frac{x}{3} \)

\[
\begin{align*}
(3) \cdot \frac{5}{7} &= \frac{x}{3} \cdot (3) \\
\frac{15}{7} &= x
\end{align*}
\]
Clearing Fractions Method

In order to clear both fractions, students will multiply both sides of the proportion by both denominators. It is important that students remember that two sides of an equation remain equal as long as the same operation is applied to both sides. They will then solve the proportion like a one-step equation.

\[
\frac{4}{9} = \frac{x}{15}
\]

\[
(9 \cdot 15) \cdot \frac{4}{9} = \frac{x}{15} \cdot (9 \cdot 15)
\]
Cross-Multiplication Method

Students can multiply opposite numerators and denominators and solve as a one-step equation. This is actually another form of multiplying both fractions by a common denominator (like the Clearing Fractions Method). The common denominator used is the product of their two denominators. In this problem, each fractional side is multiplied by the common denominator, $3x$.

\[
\frac{2}{3} \times \frac{14}{x} = 2x = 42
\]
\[
x = 21
\]

“Bat and Ball” Method

This trick is a visual of Cross-Multiplication, where students multiply the two numbers opposite one another and then divide by the remaining number. This “trick” does not promote student understanding.
Kiera can make seven bracelets in ten minutes. She read an article about a girl who made 1505 bracelets for charity, so she was curious how long it would take her to make that many without stopping.

a. Complete Kiera’s table below, and then explain how you did so. How many minutes would it take Kiera to make 1505 bracelets? How many hours?

<table>
<thead>
<tr>
<th>Bracelets</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(#)</td>
<td>(minutes)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>1505</td>
<td>?</td>
</tr>
</tbody>
</table>

b. Instead of solving this problem using a table or a graph, Kiera represented seven bracelets in ten minutes as the ratio \(\frac{10 \text{ minutes}}{7 \text{ bracelets}}\). Then she wrote an equivalent ratio as follows:

\[
\frac{10 \text{ minutes}}{7 \text{ bracelets}} = \frac{? \text{ minutes}}{1505 \text{ bracelets}}
\]

Use two different strategies to find the missing value in the equivalent ratio (Giant One, Undoing Division, Clearing Fractions, Cross-Multiplication).

Method 1:  

Method 2:
Find Someone
Who…

**Directions:** Solve the 3 proportions below using the method of your choice. Then, find 3 people who can solve the proportions 3 ways different than you. You cannot use the same person more than once.

<table>
<thead>
<tr>
<th>Problem 1:</th>
<th>Problem 2:</th>
<th>Problem 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{9}{7} = \frac{27}{x}$</td>
<td>$\frac{14}{21} = \frac{x}{7}$</td>
<td>$\frac{2x}{25} = \frac{8}{100}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>My Way:</th>
<th>My Way:</th>
<th>My Way:</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Alt. Method 1:</th>
<th>Alt. Method 1:</th>
<th>Alt. Method 1:</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Alt. Method 2:</th>
<th>Alt. Method 2:</th>
<th>Alt. Method 2:</th>
</tr>
</thead>
</table>

|----------------|----------------|----------------|
Handout 5.5: Pocket Book Instructions

Pocket Book

1. Fold a sheet of paper (8 1/2" × 11") in half like a hamburger.

2. Open the folded paper and fold one of the long sides up two inches to form a pocket. Refold along the hamburger fold so that the newly formed pockets are on the inside.

3. Glue the outer edges of the two-inch fold with a small amount of glue.

4. Optional: Glue a cover around the pocket book.
   Variation: Make a multi-paged booklet by gluing several pockets side-by-side. Glue a cover around the multi-paged pocket book.

Use 3" × 5" index cards or quarter sheets of notebook paper inside the pockets.
Store student-made books, such as two-tab books and folded books in the pockets.

Foldables are 3D Graphic Organizers created by Dinah Zike. www.dinah.com
Handout 5.6: Card Sort

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Worked</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Roses</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (Dollars)</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
</tr>
</tbody>
</table>
\[ y = 8x \quad \quad \quad y = 3x + 2 \]
\[ y = \frac{1}{2}x \quad \quad \quad y = -1x \]

A pool is drained at a rate of 3 gallons per minute.

A restaurant sells 4 slices of pizza every 10 minutes.

A video game company marks a cost of $25 per game.

Amazon charges $15.99 for every book plus shipping.
Handout 5.7: Frayer Model  
Name: __________________________________________  Date: __________

**Proportions**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Examples</th>
<th>Non-Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson 6: Solving Proportional Relationships

Focus Standard(s): 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d
Additional Standard(s): 7.RP.1
Standards for Mathematical Practice: SMP.1, SMP.6
Estimated Time: 90 minutes

Resources and Materials:
- Stop Watch
- Anchor Chart Paper
- Markers
- Handout 6.1: Pump It Up
- Unit Rates Video: https://www.youtube.com/watch?v=SpZQFKU5P70
- How to Take a Pulse: https://www.youtube.com/watch?v=W5K_HR6hxMY

Lesson Target(s):
- Students will summarize their understanding of proportionality through the Graffiti Wall.
- Students will solve problems pertaining to unit rate and proportionality.

Guiding Question(s):
- How are proportional relationships used to solve real-world and mathematical problems?
- How do you identify the constant of proportionality in tables, equations, and graphs?
## Vocabulary

### Academic Vocabulary:
- Constant of Proportionality
- Rate
- Unit Rate
- Proportional
- Ratio

### Instructional Strategies for Academic Vocabulary:
- Read and discuss the meaning of words in a mathematical context
- Write/discuss using the words

## Symbol

### Type of Text and Interpretation of Symbol

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level</th>
</tr>
</thead>
<tbody>
<tr>
<td>✚</td>
<td>Assessment (Pre-assessment, Formative, Self, or Summative)</td>
</tr>
</tbody>
</table>

## Instructional Plan

### Understanding Lesson Purpose and Student Outcomes:
Students will be assessed on their understanding of proportionality and unit rates through the Pump It Up Performance Task.

### Anticipatory Set/Introduction to the Lesson: Reflection on Proportionality
Deconstruct Graffiti Wall and return sheets to their groups. Allow teams time to add any additional information to their sheet. This provides students an opportunity to discuss the material before the performance task.

Replay the video viewed in Lesson 1: Unit Rates Video.

Give students time to work with their team to answer the question in the video using all the information they have received throughout the unit (SMP.1).

核查 Actively monitor teams and ask the following guiding questions to actively engage all students and determine student understanding:
- What representations will help you find your solution?
Activity 1: Pump It Up Performance Task

T: “Knowing about unit rate and proportionality will help you with a very important task: calculating your heart rate. You will follow the directions provided on Handout 6.1: Pump It Up to complete the activities. Then, analyze your data using tables, graphs, and equations, as well as the heart rates of others. After examining the data, you will apply reasoning when responding to questions using the multiple representations as they pertain to unit rate and proportionality.”

Show students the video How to Take a Pulse. Pair students and distribute Handout 6.1: Pump It Up. Have all students take and record their resting pulse as shown in the video. Time students as they complete each of the activities indicated on Handout 6.1: Pump It Up. Allow students 1-2 minutes of rest before starting the next activity. Ensure students are recording their heart rate after each activity. Distribute posters, markers, and Handout 6.2: Student Rubric for Performance/Culminating Task to pairs.

✓ Instruct students to complete Handout 6.1: Pump It Up (SMP.1 and SMP.6).

Note: Use the same rubric for final grading.
Reflection and Closing:
Facilitate a conversation about topics covered in the unit. Ask the following guiding questions to encourage student reflection:

- How has your understanding of proportionality grown?
- Can you teach others how to find unit rate?
- Which representation is your favorite to work with?
- How does knowing the y-intercept help you determine proportionality?
- When do we encounter proportionality in our daily lives?
- Did you have a favorite strategy for solving proportional equations for a missing value?

Homework

Students will not have homework.
Handout 6.1: Pump It Up!

Name: ___________________________                     Date: ___________

Pump It Up

After each activity, measure your pulse for 30 seconds and record your heart rate in beats per second.

Activity 1: Rest for 1 minute (Sit or lie down)  ___________ beats per second
Activity 2: Stand for 1 minute                 ___________ beats per second
Activity 3: Walk in place for 1 minute        ___________ beats per second
Activity 4: 10 squats                         ___________ beats per second
Activity 5: 25 jumping jacks                 ___________ beats per second

Use your measurements to find the unit rate for beats per minute for each activity, then complete the following tasks and create a poster to display.

Activity 1: ___________ beats per minute       Activity 2: ___________ beats per minute
Activity 3: ___________ beats per minute       Activity 4: ___________ beats per minute
Activity 5: ___________ beats per minute

On your poster:

- Create a table for each activity representing at least 5 different increments of time.
- Graph all activities on the same graph in different colors.
- Write the equation for each activity. Label with the name of the activity.
- Explain whether heart rate measurement represents a proportional relationship in terms as it relates to the situation. Label with the name of the activity.
Tables for poster:

**Activity 1:**

<table>
<thead>
<tr>
<th>Number of: _____________</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beats Per Minute</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Activity 2:**

<table>
<thead>
<tr>
<th>Number of: _____________</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beats Per Minute</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Activity 3:**

<table>
<thead>
<tr>
<th>Number of: _____________</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beats Per Minute</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Activity 4:**

<table>
<thead>
<tr>
<th>Number of: _____________</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beats Per Minute</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Activity 5:**

<table>
<thead>
<tr>
<th>Number of: _____________</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beats Per Minute</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Analyzing Heart Rates

Name: ____________________________ Date: __________

Whose heart rate are you analyzing? _______________________

1. What activity causes their heart rate to be the highest?
   
a. How do you know? Provide at least two reasons.

   b. In which representation is it easiest to see? Why?

2. What is their unit rate, in beats per minute, after 10 squats? (Include the equation in your answer)
   
a. At this rate, how long would it take their heart to beat 500 times?
3. Write and solve a proportion showing their heart rate, in beats per minute, after 25 jumping jacks and after 10 jumping jacks.

4. Complete the coordinate for each activity.

   Activity 1: (4, ___)

   Activity 2: (6, ___)

   Activity 3: (2, ___)

   Activity 4: (1, ___)

   Activity 5: (8, ___)

5. Doctors suggest that a healthy heart rate while resting for teenagers is 98 beats per minute. What is the healthy heart rate per second?

   a. Is your heart rate above or below the healthy resting heart rate?

   b. Explain how you know.
<table>
<thead>
<tr>
<th>Novice (1)</th>
<th>Tables</th>
<th>Graphs</th>
<th>Equations</th>
<th>Reasoning and explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) I created 1-2 tables.</td>
<td>a) I graphed 1-2 situations.</td>
<td>a) I wrote 1-2 equations.</td>
<td>a) I provided little explanation.</td>
<td></td>
</tr>
<tr>
<td>Apprentice (2)</td>
<td>a) I created 3 tables. b) The tables had minimal errors.</td>
<td>a) I graphed 3 situations b) The graph was scaled correctly but difficult to read.</td>
<td>a) I wrote 3 equations. b) I showed little evidence of how to use an equation while answering questions.</td>
<td>a) I provided some explanation. b) I used few examples when explaining.</td>
</tr>
<tr>
<td>Practitioner (3)</td>
<td>a) I created 4 tables b) The tables had minimal errors. c) The tables were clear and precise.</td>
<td>a) I graphed 4 situations. b) The graph was scaled correctly. c) I used my graph correctly when answering questions about coordinates.</td>
<td>a) I wrote 4 equations. b) I used equations to support my answers. c) I used equations to correctly answer unit rate questions.</td>
<td>a) I provided explanations. b) I used examples. c) I reasoned and compared unit rates.</td>
</tr>
<tr>
<td>Expert (4)</td>
<td>a) I created 5 tables. b) The tables had no mistakes. c) Tables were clear and precise. d) I used the coordinates from the table to correctly make ratios and justify answers.</td>
<td>a) I graphed 5 situations. b) My graph was scaled correctly. c) I used my graph correctly when answering questions about coordinates. d) I used my graph correctly when comparing unit rates to determine proportionality.</td>
<td>a) I wrote 5 equations. b) I used equations to correctly justify answers. c) I used equations to correctly find unit rate. d) I correctly solved equations when given values for the variable.</td>
<td>a) I provided clear explanations. b) I used strong examples. c) I reasoned and compared unit rates to determine proportionality. d) I supported my answers with ratios, tables, and equations.</td>
</tr>
</tbody>
</table>
For training or questions regarding this unit, please contact:

exemplarunit@mdek12.org