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Multistate Standard-Setting Technical Report

PRAXIS™ MATHEMATICS: CONTENT KNOWLEDGE (5161)

Licensure and Credentialing Research

ETS

Princeton, New Jersey

March 2013

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EXECUTIVE SUMMARY

To support the decision-making process of education agencies establishing a passing score (cut score) for the Praxis™ Mathematics: Content Knowledge (5161) test, research staff from Educational Testing Service (ETS) designed and conducted a multistate standard-setting study.

PARTICIPATING STATES

Panelists from 24 states and Washington, DC were recommended by their respective education agency. The education agencies recommended panelists with (a) experience either as mathematics teachers or college faculty who prepare mathematics teachers and (b) familiarity with the knowledge and skills required of beginning mathematics teachers.

RECOMMENDED PASSING SCORE

ETS provides a recommended passing score from the multistate standard-setting study to help education agencies determine an appropriate operational passing score. For the Praxis Mathematics: Content Knowledge test, the recommended passing score[1] is 32 out of a possible 50 raw-score points. The scaled score associated with a raw score of 32 is 160 on a 100–200 scale.

[1] Results from the two panels participating in the study were averaged to produce the recommended passing score.

To support the decision-making process for education agencies establishing a passing score (cut score) for the Praxis™ Mathematics: Content Knowledge (5161) test, research staff from ETS designed and conducted a multistate standard-setting study in February 2013 in Princeton, New Jersey. Education agencies[2] recommended panelists with (a) experience, either as mathematics teachers or college faculty who prepare mathematics teachers and (b) familiarity with the knowledge and skills required of beginning mathematics teachers. Twenty-four states and Washington, DC(see Table 1) were represented by 35 panelists. (See Appendix A for the names and affiliations of the panelists.)

Table 1

Participating Jurisdictions and Number of Panelists

Alaska (1 panelist)	North Dakota (1 panelist)
Arkansas (2 panelists)	Pennsylvania (1 panelist)
Delaware (1 panelist)	Rhode Island (1 panelist)
Idaho (2 panelists)	South Carolina (1 panelist)
Kentucky (1 panelist)	South Dakota (1 panelist)
Louisiana (1 panelist)	Tennessee (1 panelist)
Maine (1 panelist)	Utah (2 panelists)
Maryland (2 panelists)	Vermont (1 panelist)
Mississippi (2 panelists)	Washington, DC (1 panelist)
Nevada (1 panelist)	Wisconsin (2 panelists)
New Hampshire (2 panelists)	West Virginia (1 panelist)
New Jersey (2 panelists)	Wyoming (2 panelists)
North Carolina (2 panelists)	

The following technical report contains three sections. The first section describes the content and format of the test. The second section describes the standard-setting processes and methods. The third section presents the results of the standard-setting study.

ETS provides a recommended passing score from the multistate standard-setting study to education agencies. In each jurisdiction, the department of education, the board of education, or a designated educator licensure board is responsible for establishing the operational passing score in

[2] States and jurisdictions that currently use Praxis were invited to participate in the multistate standard-setting study.

accordance with applicable regulations. This study provides a recommended passing score,[3] which represents the combined judgments of two panels of experienced educators. Each jurisdiction may want to consider the recommended passing score but also other sources of information when setting the final Praxis Mathematics: Content Knowledge passing score (see Geisinger & McCormick, 2010). A jurisdiction may accept the recommended passing score, adjust the score upward to reflect more stringent expectations, or adjust the score downward to reflect more lenient expectations. There is no *correct* decision; the appropriateness of any adjustment may only be evaluated in terms of its meeting the jurisdiction's needs.

Two sources of information to consider when setting the passing score are the standard error of measurement (SEM) and the standard error of judgment (SEJ). The former addresses the reliability of the Praxis Mathematics: Content Knowledge test score and the latter, the reliability of panelists' passing-score recommendation. The SEM allows a jurisdiction to recognize that any test score on any standardized test—including a Praxis Mathematics: Content Knowledge test score—is not perfectly reliable. A test score only *approximates* what a candidate truly knows or truly can do on the test. The SEM, therefore, addresses the question: How close of an approximation is the test score to the *true* score? The SEJ allow a jurisdiction to gauge the likelihood that the recommended passing score from a particular panel would be similar to the passing scores recommended by other panels of experts similar in composition and experience. The smaller the SEJ, the more likely that another panel would recommend a passing score consistent with the recommended passing score. The larger the SEJ, the less likely the recommended passing score would be reproduced by another panel.

In addition to measurement error metrics (e.g., SEM, SEJ), each jurisdiction should consider the likelihood of classification error. That is, when adjusting a passing score, policymakers should consider whether it is more important to minimize a false-positive decision or to minimize a false-negative decision. A false-positive decision occurs when a candidate's test score suggests he should receive a license/certificate, but his actual level of knowledge/skills indicates otherwise (i.e., the candidate does not possess the required knowledge/skills). A false-negative decision occurs when a candidate's test score suggests that she should not receive a license/certificate, but she actually does possess the required

[3] In addition to the recommended passing score averaged across the two panels, the recommened passing scores for each panel are presented.

knowledge/skills. The jurisdiction needs to consider which decision error may be more important to minimize.

Overview of the Praxis Mathematics: Content Knowledge Test

The Praxis Mathematics: Content Knowledge *Test at a Glance* document (ETS, in press) describes the purpose and structure of the test. In brief, the test measures whether entry-level mathematics teachers have the knowledge/skills believed necessary for competent professional practice.

The two-hour assessment contains 60 selected-response and numeric-entry items[4] covering two content areas: *Number and Quantity, Algebra, Functions, and Calculus* (approximately 41 items) and *Geometry, Probability and Statistics, and Discrete Mathematics* (approximately 19 items).[5] The reporting scale for the Praxis Mathematics: Content Knowledge test ranges from 100 to 200 scaled-score points.

Processes and Methods

The design of the standard-setting study included two, independent expert panels. Before the study, panelists received an email explaining the purpose of the standard-setting study and requesting that they review the content specifications for the test. This review helped familiarize the panelists with the general structure and content of the test.

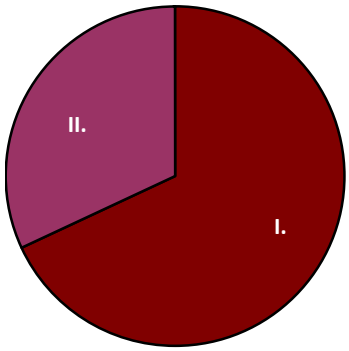
For each panel, the standard-setting study began with a welcome and introduction by the meeting facilitator. The facilitator described the test, provided an overview of standard setting, and presented the agenda for the study. Appendix B shows the agenda for the panel meeting.

[4] Ten of the 60 selected-response and numeric-entry items are pretest items and do not contribute to a candidate's score.
[5] The number of items for each content area may vary slightly from form to form of the test.

Mathematics: Content Knowledge (5161)

Test at a Glance

Test Name	**Mathematics: Content Knowledge**
Test Code	**5161**
Time	**150 Minutes**
Number of Questions	**60 Selective Response Questions**
Format	**Innovative Multiple-choice**



Content Categories	Approximate Number of Questions	Approximate Percent of Examination
I. Number and Quantity, Algebra, Functions, and Calculus	41	68%
II. Geometry, Probability and Statistics, and Discrete Mathematics	19	32%

About This Test

The Mathematics: Content Knowledge test measures whether entry-level mathematics educators have the standards relevant knowledge skills, and abilities believed necessary for competent professional practice.

This test may contain some questions that will not count toward your score.

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Topics Covered

I. Number and Quantity, Algebra, Functions, and Calculus

A. Number and Quantity

- Understands the properties of exponents.
  - — Perform operations involving exponents, including negative and rational exponents.
  - — Demonstrate an understanding of the properties of exponential expressions.
  - — Use the properties of exponents to rewrite expressions that have radicals or rational exponents.

- Understands the properties of rational and irrational numbers, and the interactions between those sets of numbers.
  - — Recognize that the sum or product of two rational numbers is rational.
  - — Recognize that the sum of a rational number and an irrational number is irrational.
  - — Recognize that the product of a nonzero rational number and an irrational number is irrational.
  - — Recognize that the sum or product of two irrational numbers can be rational or irrational.

- Understands how to solve problems by reasoning quantitatively (e.g., dimensional analysis, reasonableness of solutions).
  - — Use units as a way to understand problems and to guide the solution of multistep problems.
  - — Choose and interpret units consistently in formulas.
  - — Choose and interpret the scale and the origin in graphs and data displays.
  - — Recognize the reasonableness of results within the context of a given problem.

- Understands the structure of the natural, integer, rational, real, and complex number systems and how the basic operations (+, –, ×, and ÷) on numbers in these systems are performed.
  - — Solve problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers.
  - — Apply the order of operations.
  - — Given operations on a number system, determine whether the properties (e.g., commutative, associative, distributive) hold.
  - — Compare, classify, and order real numbers.
  - — Simplify and approximate radicals.
  - — Find conjugates of complex numbers.
  - — Demonstrate an understanding of the properties of counting numbers (e.g., prime, composite, prime factorization, even, odd, factors, multiples).

- Understands how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions.
  - — Solve quadratic equations with real coefficients that have complex solutions.
  - — Extend polynomial identities to the complex numbers (e.g., x2 + y2 = (x + yi)(x – yi)).
  - — Verify the fundamental theorem of algebra for quadratic polynomials.

- Understands how to perform operations on matrices and how to use matrices in applications.
  - — Use matrices to represent and manipulate data.
  - — Multiply matrices by scalars to produce new matrices.
  - — Add, subtract, and multiply matrices of appropriate dimensions.

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- Understand that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties.
- Understand the role played by zero, and identity matrices in matrix addition and multiplication.
- Understand that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

- Understands how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions.
  - Apply the concept of a ratio and use ratio language and notation to describe a relationship between two quantities.
  - Compute unit rates.
  - Use ratio reasoning to convert rates.
  - Solve problems involving scale factors.
  - Recognize and represent proportional and inversely proportional relationships between two quantities.
  - Use proportional relationships to solve multistep ratio, average, and percent problems.
  - Solve measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate.
  - Convert units within the metric and customary systems.

- Knows how to analyze both precision and accuracy in measurement situations.
  - Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
  - Calculate or estimate absolute and relative error in the numerical answer to a problem.
- Understands various ways to represent and compare very large and very small numbers (e.g., scientific notation, orders of magnitude).
  - Represent and compare very large and very small numbers.

- Understands how to both estimate and perform calculations on very large and very small quantities.
  - Use orders of magnitude to estimate very large and very small numbers.
  - Perform calculations on numbers in scientific notation.

B. Algebra

- Understands how to write algebraic expressions in equivalent forms.
  - Use the structure of an expression to identify ways to rewrite it.
  - Understand how to rewrite quadratic expressions for specific purposes (e.g., factoring/finding zeros, completing the square/finding maxima or minima).
  - Use the properties of exponents to rewrite expressions for exponential functions.

- Understands how to perform arithmetic operations on polynomials.
  - Add, subtract, and multiply polynomials.

- Understands the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions.
  - Know and apply the remainder theorem: for a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
  - Use factorization to identify zeros of polynomials.

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- Use zeros of a polynomial to construct a rough graph of the function defined by the polynomial.

- Understands how to use polynomial identities (e.g., difference of squares, sum and difference of cubes) to solve problems.
  - Apply the binomial theorem for the expansion of (x + y)n in powers of x and y for a positive integer n.

- Understands how to rewrite rational expressions and perform arithmetic operations on rational expressions.
  - Rewrite simple rational expressions in different forms.
  - Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
  - Add, subtract, multiply, and divide rational expressions.

- Understands how to create equations and inequalities that describe relationships.
  - Create equations and inequalities in one variable and use them to solve problems and graph solutions on the number line.
  - Create equations and inequalities in two or more variables to represent relationships between quantities, solve problems, and graph them on the coordinate plane with labels and scales.
  - Represent constraints by equations, inequalities, or systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
  - Rearrange formulas to highlight a quantity of interest (e.g., solve d = rt for t).

- Understands how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions.
  - Explain each step in solving a simple equation.
  - Solve simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions.

- Understands how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities in one variable.
  - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Use the method of completing the square to transform any quadratic equation in x into the equivalent form $(x – p)2 = q$.
  - Solve equations using a variety of methods (e.g., using graphs, using the quadratic formula, or factoring).
  - Use different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation.
  - Write complex solutions in the form a ± bi.

- Understands how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities.
  - Explain why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system.
  - Solve a system consisting of two linear equations in two variables algebraically and graphically.
  - Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

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- Represent a system of linear equations as a single matrix equation.
- Find the inverse of a matrix if it exists, and use it to solve systems of linear equations.
- Explain why the x-coordinates of the intersection points of the graphs of y = f(x) and y = g(x) are the solutions of f(x) = g(x).
- Find the solutions of f(x) = g(x) approximately (e.g., use technology to graph the functions, make tables of values, find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, or logarithmic functions.
- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

- Understands the properties of number systems under various operations.
  - Given operations on algebraic expressions, determine whether the properties hold (e.g., commutative, associative, distributive).

- Understands the concept of rate of change of nonlinear functions.
  - Calculate and interpret the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval.

- Understands the concepts of intercept(s) of a line and slope as a rate of change.
  - Calculate and interpret the intercepts of a line.
  - Calculate and interpret the slope of a line presented symbolically, numerically, or graphically.
  - Estimate the rate of change of a linear function from a graph.

- Understands how to find the zero(s) of functions.
  - Uses a variety of techniques to find and analyze the zero(s) (real and complex) of functions.

C. Functions

- Understands the function concept and the use of function notation.
  - Recognize that functions are sets of ordered pairs.
  - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Use function notation, evaluate functions, and interpret statements that use function notation in terms of a context.
  - Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

- Understands how to find the domain and range of a function and a relation.
  - Identify the domain and range of a function or relation.
  - Determine the domain of a function from a function rule (e.g., f(x) = 2x + 1), graph, set of ordered pairs, or table.

Understands how function behavior is analyzed using different representations. (e.g., graphs, mappings, tables).

- For a function that models a relationship between two quantities, interpret key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities.
- Given a verbal description of a relation, sketch graphs that show key features of that relation.

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- Graph functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically and identify key features of the graph.
- Write a function that is defined by an expression in different but equivalent forms to reveal different properties of the function (e.g., zeros, extreme values, symmetry of the graph).
- Interpret the behavior of exponential functions (e.g., growth, decay).
- Understand how to determine if a function is odd, even, or neither and any resulting symmetries.

- Understands how functions and relations are used to model relationships between quantities.
  - Write a function that relates two quantities.
  - Determine an explicit expression or a recursive process that builds a function from a context.

- Understands how new functions are obtained from existing functions (e.g., compositions, transformations, inverses).
  - Describe how the graph of g(x) is related to the graph of f(x), where g(x) = f(x) + k, g(x) = k f(x), g(x) = f(kx), or g(x) = f(x + k) for specific values of k (both positive and negative), and find the value of k given the graphs.
  - Determine if a function has an inverse and write an expression for the inverse.
  - Verify by composition if one function is the inverse of another.
  - Given that a function f has an inverse, find values of the inverse function from a graph or a table of f.
  - Given a noninvertible function, determine a largest possible domain of the function that produces an invertible function.
  - Understand the inverse relationship between exponential and logarithmic functions and use this relationship to solve problems.
  - Combine standard function types using arithmetic operations.
  - Perform domain analysis on functions resulting from arithmetic operations.
  - Compose functions algebraically, numerically, and graphically.
  - Perform domain analysis on functions resulting from compositions.

- Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems.
  - Understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
  - Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).
  - Observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
  - Express the solution to an exponential equation with base b as a logarithm (e.g., $3 \cdot 25t = 20$, $3 \cdot e5t = 20$).
  - Use technology to evaluate logarithms that have any base.
  - Interpret the parameters in a linear or exponential function in terms of a context (e.g., A(t) = Pert).
  - Use quantities that are inversely related to model phenomena.

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- Understands how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains.
  - — Understand radian measure (e.g., 1 radian is the measure of a central angle that subtends an arc with length equal to the length of the radius).
  - — Understand how the domains of trigonometric functions can be extended beyond 0 to 2π using the unit circle.
  - — Use special triangles (i.e., 30-60-90, 45-45-90) to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.
  - — Use reference angles to find the values of trigonometric functions at angles outside the interval 0 to $\frac{\pi}{2}$.
  - — Use the unit circle to explain symmetry and periodicity of trigonometric functions.

- Understands how periodic phenomena are modeled using trigonometric functions.
  - — Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
  - — Understand how to restrict the domain of a trigonometric function so that its inverse can be constructed.
  - — Use inverse functions to solve trigonometric equations that arise in modeling contexts, and interpret them in terms of the context.

- Understands the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, and difference of angles).
  - — Use Pythagorean identities (e.g., sin2 θ + cos2 θ = 1).
  - — Use trigonometric identities to rewrite expressions and solve equations.
  - — Understand trigonometric identities in the context of equivalent graphs of trigonometric functions (e.g., y = sin x and y = cos ($\frac{\pi}{2}$ – x) are equivalent graphs).
  - — Prove Pythagorean identities (e.g., sin2 θ + cos2 θ = 1).

- Knows how to interpret representations of functions of two variables (e.g., three-dimensional graphs, tables)..
  - — Interpret representations of functions of two variables.

- Understands how to solve equations (e.g., trigonometric, logarithmic, exponential).
  - — Solve trigonometric, logarithmic, and exponential equations.

D. Calculus

- Understands the meaning of a limit of a function and how to calculate limits of functions, determine when the limit does not exist, and solve problems using the properties of limits.
  - — Graphically analyze the limit of f(x) as x approaches a fixed value from both left and right.
  - — Solve limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that x is approaching.
  - — Analyze one-sided limits for various functions to see whether or not the limit exists.
  - — Recognize limits that do not exist, such as $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ and $\lim_{x\to 0} \frac{1}{\sqrt[3]{x^2}}$.

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- Understands the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change.
  - — Construct a function graph for a given function and a given point (a, f(a)), and explain what happens to the succession of slopes of secant lines connecting (a, f(a)) to (x, f(x)) as x approaches a, from both the right side and the left side.
  - — State the limit definition of the derivative, and use it to find the derivative of a given function at a given value of x and to find the derivative function.

- Understands how to show that a particular function is continuous.
  - — Apply the three steps (i.e., $f(a)$ exists, $\lim_{x \to a} f(x)$ exists, and $f(a) = \lim_{x \to a} f(x)$) that are part of the definition of what it means for a function to be continuous at x = a to verify whether a given function is continuous at a given point.

- Knows the relationship between continuity and differentiability.
  - — Give examples of functions that are continuous at x = a but not differentiable at x = a, and explain why.

- Understands how to approximate derivatives and integrals numerically.
  - — Given a table of values, use the slope of a secant line to approximate a derivative.
  - — Use the midpoint rule, trapezoid rule, or other Reimann sums to find numerical approximations for integrals.

- Understands how and when to use standard differentiation and integration techniques.
  - — Use standard differentiation techniques.
  - — Use standard integration techniques.
  - — Understand the relationship between position, velocity, and acceleration functions of a particle in motion.

- Understands how to analyze the behavior of a function (e.g., extrema, concavity, symmetry).
  - — Use the first and second derivatives to analyze the graph of a function.

- Understands how to apply derivatives to solve problems (e.g., related rates, optimization).
  - — Apply derivatives to solve problems.

- Understands the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem).
  - — Solve problems using the foundational theorems of calculus.
  - — Understand the relationship between differentiation and integration, including the role of the fundamental theorems of calculus.
  - — Match graphs of functions with graphs of their derivatives or accumulations.
  - — Understand how to use differentiation and integration of a function to express rates of change and total change.
  - — Understand and calculate the average value of a function over an interval (i.e., mean value theorem of integrals).

- Understands integration as a limit of Riemann sums.
  - — Calculate a definite integral using a limit of Riemann sums.

- Understands how to use integration to compute area, volume, distance, or other accumulation processes.
  - — Use integration techniques to compute area, volume, distance, or other accumulation processes.

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- Knows how to determine the limits of sequences, if they exist.
  - — Determine the limits of sequences when they exist.

- Is familiar with simple infinite series.
  - — Determine if simple infinite series converge or diverge.
  - — Find the sum of a simple infinite series if it exists.
  - — Find the partial sum of a simple infinite series.

II. Geometry, Probability and Statistics, and Discrete Mathematics

A. Geometry

- Understands transformations in a plane.
  - — Know precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines.
  - — Represent transformations in the plane.
  - — Describe transformations as functions that take points in the plane as inputs, and give other points as outputs.
  - — Recognize whether a transformation preserves distance and angle measure.
  - — Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that map it onto itself.
  - — Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
  - — Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.
  - — Specify a sequence of transformations that will map a given figure onto another figure.

- Understands how to prove geometric theorems such as those about lines and angles, triangles, and parallelograms.
  - — Prove theorems about lines and angles.
  - — Prove theorems about triangles.
  - — Prove theorems about parallelograms.

- Understands how geometric constructions are made with a variety of tools and methods.
  - — Recognize formal geometric constructions.
  - — Explain how formal geometric constructions are made (e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle).

- Understands congruence and similarity in terms of transformations.
  - — Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.
  - — Verify the properties of dilations given by a center and a scale factor.
  - — Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
  - — Given two figures, use the definition of similarity in terms of dilations to decide if the figures are similar.
  - — Explain how the criteria for triangle congruence (e.g., ASA, SAS, HL) follow from the definition of congruence in terms of rigid motions.
  - — Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
  - — Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

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- Understands how trigonometric ratios are defined in right triangles.
  - — Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
  - — Explain and use the relationship between the sine and cosine of complementary angles.
  - — Use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.

- Understands how trigonometry is applied to general triangles.
  - — Derive the formula A = $\frac{1}{2}$ ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side and use it to solve problems.
  - — Apply the law of sines and the law of cosines to find unknown measurements in triangles.

- Understands and applies theorems about circles.
  - — Identify and describe relationships among inscribed angles, radii, and chords.
  - — Prove properties of angles for a quadrilateral inscribed in a circle.
  - — Construct a tangent line from a point outside a given circle to the circle.

- Understands arc length and area measurements of sectors of circles.
  - — Derive and use the fact that the length of the arc intercepted by a central angle is proportional to the circumference.
  - — Derive and use the formula for the area of a sector.

- Knows how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section.
  - — Derive and use the equation of a circle of given center and radius.
  - — Complete the square to find the center and radius of a circle given by an equation in standard form.
  - — Derive the equation of a parabola given a focus and directrix.
  - — Derive and use the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant.

- Understands how to use coordinate geometry to algebraically prove simple geometric theorems.
  - — Use coordinates to prove simple geometric theorems algebraically.
  - — Prove the slope criteria for parallel and perpendicular lines, and use parallel and perpendicular lines to solve geometric problems.
  - — Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
  - — Use coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals.

- Understands how perimeter, area, surface area, and volume formulas are used to solve problems.
  - — Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
  - — Use the perimeter and area of geometric shapes to solve problems.
  - — Use the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems.

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- Knows how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects.
  - Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
  - Use two-dimensional representations of three-dimensional objects to visualize and solve problems.

- Knows how to apply geometric concepts in real-world situations.
  - Use geometric shapes, their measures, and their properties to describe objects.
  - Apply concepts of density based on area and volume in modeling situations.
  - Apply geometric methods to solve design problems.

- Understands the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving. .
  - Solve problems involving parallel, perpendicular, and intersecting lines.
  - Apply angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems.
  - Solve problems that involve medians, midpoints, and altitudes.
  - Solve problems involving special triangles (e.g., isosceles, equilateral, right).
  - Know geometric properties of various quadrilaterals (e.g., parallelograms, trapezoids).
  - Know relationships among quadrilaterals.
  - Solve problems involving angles and diagonals.
  - Solve problems involving polygons with more than four sides.

B. Probability and Statistics

- Understands how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions).
  - Represent data with plots on the real number line (e.g., dot plots, histograms, and box plots).
  - Use statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets.
  - Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers.
  - Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate.
  - Estimate areas under the normal curve.

- Understands how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series).
  - Summarize and interpret categorical data for two categories in two-way frequency tables (e.g., joint, marginal, conditional relative frequencies).
  - Recognize possible associations and trends in the data.
  - Represent data for two quantitative variables on a scatterplot, and describe how the variables are related.

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- Understands how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient).
  - Use technology to fit a function to data (i.e., linear regression).
  - Use functions fitted to data to solve problems in the context of the data.
  - Assess the fit of a function by plotting and analyzing residuals.
  - Interpret the slope and the intercept of a regression line in the context of the data.
  - Compute and interpret a correlation coefficient.
  - Distinguish between correlation and causation.

- Understands statistical processes and how to evaluate them.
  - Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
  - Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation).

- Understands how to make inferences and justify conclusions from samples, experiments, and observational studies.
  - Recognize the purposes of and differences among samples, experiments, and observational studies, and explain how randomization relates to each.
  - Use data from a sample to estimate a population mean or proportion.
  - Use data from a randomized experiment to compare two treatments.
  - Use results of simulations to decide if differences between parameters are significant.
  - Evaluate reports based on data.

- Understands the concepts of independence and conditional probability and how to apply these concepts to data.
  - Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.
  - Understand that two events, A and B, are independent if and only if $P(A \cap B) = P(A)P(B)$.
  - Understand the conditional probability of A given B as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of A and B as saying that $P(A|B) = P(A)$ and $P(B|A) = P(B)$.
  - Recognize and explain the concepts of conditional probability and independence.

- Understands how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities.
  - Calculate probabilities of simple and compound events.
  - Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
  - Find $P(A|B)$, and interpret it in terms of a given model.
  - Apply the addition rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret it in terms of a given model.
  - Apply the general multiplication rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret it in terms of a given model.
  - Calculate probabilities using the binomial probability distribution.

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- Knows how to make informed decisions using probabilities and expected values.
  - Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graph the corresponding probability distribution using the same graphical displays as for data distributions.
  - Calculate the expected value of a random variable, and interpret it as the mean of the probability distribution.
  - Develop a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and find the expected value.
  - Develop a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and find the expected value.
  - Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
  - Analyze decisions and strategies using probability concepts (e.g., fairness).

- Understands how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions.
  - Given the results of simulations, construct experimental probability distributions.
  - Given the results of simulations, make informal inferences about theoretical probability distributions.

- Understands how to find probabilities involving finite sample spaces and independent trials.
  - Use the fundamental counting principle to find probabilities involving finite sample spaces and independent trials.

C. Discrete Mathematics

- Understands sequences (e.g., arithmetic, recursively defined, geometric)..
  - Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
  - Evaluate, extend, or algebraically represent rules that involve number patterns.
  - Explore patterns in order to make conjectures, predictions, or generalizations.

- Is familiar with how recursion can be used to model various phenomena.
  - Find values of functions defined recursively, and understand how recursion can be used to model various phenomena.
  - Convert between recursive and closed-form expressions for a function, where possible.

- Has knowledge of equivalence relations.
  - Determine whether a binary relation on a set is reflexive, symmetric, or transitive.
  - Determine whether a relation is an equivalence relation.

- Understands the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena.
  - Understand the differences between discrete and continuous representations (e.g., data, functions).
  - Understand how discrete and continuous representations can be used to model various phenomena.

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- Understands basic terminology and symbols of logic.
    - Understand the basic terminology of logic.
    - Understand the symbols of logic.
    - Use logic to evaluate the truth of statements.
    - Use logic to evaluate the equivalence of statements (e.g., statement and contra positive).

- Understands how to use counting techniques such as the multiplication principle, permutations, and combinations.
    - Use counting techniques to solve problems.

- Understands basic set theory (e.g., unions, differences, Venn diagrams).
    - Solve problems using basic set theory (i.e., union, intersection, complement, difference).
    - Use Venn diagrams to answer questions about sets.

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