“Cuisenaire Trains”

5.NF.7a & 5.NF.7c

Spring 2021
As we continue our efforts to develop high-quality instructional materials (HQIM) and resources, the Mississippi Department of Education (MDE), through the Academic Education Office, would like to showcase instructional practices and activities that foster conceptual understanding through the use of manipulatives in the mathematics classroom.

The *Mississippi Mathematics Manipulatives Manual* features activities meant to serve as short, hands-on procedures that may be implemented before, during, or after a lesson to support the teaching and learning process of the Mississippi College- and Career-Readiness Standards (MCCRS) for Mathematics. Alignment with the MCCRS Scaffolding Document has been included for additional support. Teachers may contact staff at the MDE if they would like to borrow manipulatives for classroom use.

Teachers may modify these activities to meet the needs of the students they serve and their instructional delivery model (virtual, in-person, or hybrid).

**Special Thanks:**
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Cuisenaire Trains

**MANIPULATIVE(S):**
- Cuisenaire Rods

**GRADE LEVEL OR COURSE TITLE:**
CCR Mathematics Grade 5

**DOMAIN AND CLUSTER HEADING:**
Number and Operations – Fractions (NF)
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

**STANDARD(S):**

5.NF.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for \((1/3) \div 4\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \((1/3) \div 4 = 1/12\) because \((1/12) \times 4 = 1/3\).

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

*Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

**PREREQUISITE SKILLS:**
- How to divide whole numbers and what it means to divide using the partitioning and repeated subtraction models.
- Division is the inverse of multiplication.
- How to use visual models to divide whole numbers with and without remainders.
- How to contextualize division problems using whole numbers.
- Division is not a commutative operation.
- Unit fractions have a numerator of one and can be combined to create non-unit fractions.
ACTIVITY:  

Note: Activity Sheet Attached

1. In whole group discussion, hold up a purple Cuisenaire rod and tell students that this rod represents \( \frac{1}{2} \) and that they will explore what would “fit inside” the purple rod (i.e., how many red pieces would be the same as one purple = \( \frac{1}{2} \)).

2. Show students this Cuisenaire Train image, as shown in Figure 1 below, or build it and display it for students: one purple rod, then two red rods stacked atop the purple rod, then four white rods stacked atop the red rods. Tell students that you started with a purple rod and that you “partitioned” it into equal parts in two different ways.

\[
\begin{align*}
&1- \text{ purple rod } (1/2) = 2- \text{ red rods } (1/4) = 4- \text{ white rods } (1/8) \\
\end{align*}
\]

3. Ask students “If the purple rod is worth \( \frac{1}{2} \), what is the value of one red rod? What is the value of one white rod? Allow students to turn and explain their thinking to a partner.

4. After group discussion, record students’ thinking in a table (as shown in figure 2). Ask students if they notice any patterns emerging in the table when they partition a unit fraction.

<table>
<thead>
<tr>
<th>If I start with a _____ (color) rod and I assume it’s worth _____ ...</th>
<th>Then I can partition it equally into _____ (how many?) (color) rods ......</th>
<th>So, each of these rods is worth ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>purple = ( \frac{1}{2} )</td>
<td>2 red</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>purple = ( \frac{1}{2} )</td>
<td>4 white</td>
<td>( \frac{1}{8} )</td>
</tr>
</tbody>
</table>

5. Next, pair students with a partner and provide each student with a copy of the Cuisenaire Trains Recording Sheet. (Attached)
6. Ensure each student has access to a set of Cuisenaire rods. **Note:** For a virtual option visit [https://mathsbot.com/manipulatives/rods](https://mathsbot.com/manipulatives/rods) (For an alternative, virtual option see the Resources section below.)

7. Have students select a rod (color) of their choice to represent one of the following unit fractions: \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \) or \( \frac{1}{8} \). Students should record this starting point/unit fraction in the first column of the Cuisenaire Trains Recording Sheet.

8. Students will then explore how to partition the unit fraction into equal parts using another color Cuisenaire rod. After that, students will work to find the value of each of the smaller rods. *(For example, as modeled in Figure 3, if a dark green rod is chosen to represent \( \frac{1}{3} \), a student may conclude that one dark green rod can be partitioned into 2 light green rods, in which case each light green rod would represent \( \frac{1}{6} \).)*

9. Students should continue to explore other ways to partition the same unit fraction. *(For example, also modeled in Figure 3, the same dark green piece representing \( \frac{1}{3} \) could be partitioned into six white rods: each representing \( \frac{1}{18} \)).*

10. Students continue to complete the Cuisenaire Trains Recording Sheet by repeating steps #7-9 above. Each time choosing different rods to represent a different unit fraction and then finding as many ways as possible to partition that unit fraction with other Cuisenaire rods of the same color.

**Note:** Students may use the back of their Cuisenaire Trains Recording Sheet to sketch the trains they build to model their thinking or as a tool for figuring out the value of each rod piece.

**QUESTIONS TO CONSIDER:**

- What are your observations?
- What patterns do you see?
- How could you use Cuisenaire rods to determine whether the patterns you see are always true?
• How did you figure out what each rod was worth? What strategies did you develop?
• What patterns (across the rows and between the rows) did you notice in your table on the Playing with Cuisenaire Rods Train Recording Sheet?
• Which rods had the most solutions? Which rods had the fewest?
• Which unit fractions were the easiest to start with? Explain.
• How can you use math symbols to record what is happening across the rows?
• What is happening when you take a fraction and divide it by a whole number?
• How does the quotient’s value compare to the dividend and divisor (i.e., when the dividend is a unit fraction, and the divisor is a whole number)? Is it smaller than both? Larger than both? Larger/smaller than the dividend? Larger/smaller than the divisor? Explain.

RESOURCES:

- [Mississippi Mathematics Scaffolding Document](#) (Grade 5, Pages 36-37, 40-41)
- [2016 MCCRS for Mathematics](#)
- [Virtual Cuisenaire Rods](#) - MathsBot
- [Modeling Fractions with Cuisenaire Rods](#) - PBS.Panda
- [Cuisenaire Trains, Mindset Mathematics Grade 5: Visualizing and Investigating Big Ideas](#) by Jo Boaler, Jen Munson, & Cathy Williams (Jossey-Bass, 2018)
- Playing with Cuisenaire Trains Recording Sheet, *Mindset Mathematics, Grade 5* (pg. 216)

Optional: The University of Mississippi’s Center for Mathematics and Science Education has an extensive inventory of math (and science and technology) tools and manipulatives that teachers may borrow for classroom use at no charge. Click the link below to access the inventory list and complete a check-out request.

- [CMSE Manipulatives](#)

BEYOND THE ACTIVITY:

• **Extension:** Have students investigate whether the patterns they observed would hold true for all unit fractions? non-unit proper fractions? improper fractions?

• **Misconceptions:** Throughout this activity, academic vocabulary is important. It is helpful to use the word “partition” to refer to dividing one of the rods into smaller sized rods. Listen for students who might say “I divided 4 by ½” when they actually mean “I divided ½ by 4”. For the first time, students must consider the order of division. Also, listen for students using the word “into”. For example: “I divided 4 into 16” when they mean 16/4. Precision of language will help students build conceptual understanding and maintain models of what is happening and describe it accurately.
**Activity Sheet**

Cuisenaire Trains Recording Sheet

Student Name ____________________________________________

<table>
<thead>
<tr>
<th>If I start with a ____ color rod, and assume it’s worth ____...</th>
<th>and I can partition it equally into ___ (color) rods...</th>
<th>then, each of rod is worth...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example: Purple = $\frac{1}{2}$</td>
<td>2-Red</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>

Recreated from “Playing with Cuisenaire Rods Recording Sheet”, Mindset Mathematics, Grade 5, copyright ©2018 by Jo Boaler, Jen Munson, Cathy Williams