

EXEMPLAR Units & Lessons MATHEMATICS Algebra I

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MISSISSIPPI DEPARTMENT OF EDUCATION

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Introduction

Mission Statement

The Mississippi Department of Education (MDE) is dedicated to student success, including the improvement of student achievement in English Language Arts (ELA) and mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a global society. The Mississippi College- and Career-Readiness Standards (MS CCRS) provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level or course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy. The goal of the MDE is to provide educators with the training and resources to understand and implement the MS CCRS effectively.

Purpose

In efforts to facilitate implementation and promote understanding of the MS CCRS for ELA and mathematics, the W. K. Kellogg Foundation generously awarded the MDE a grant to secure a cadre of effective educators to develop the MS CCRS Exemplar Units for teachers. Specifically, a group of highly-effective Mississippi educators developed exemplar instructional units and lessons aligned to the MS CCRS for ELA and mathematics. The MS CCRS Exemplar Units address difficult-to-teach standards as determined by teachers and are designed to serve as exemplar models for instructional units, lessons, and resources. The MS CCRS Exemplar Units have been vetted through nationally renowned vendors to ensure exemplar quality.

Design Overview

The MS CCRS Exemplar Units for ELA and mathematics address grade-level specific standards for Pre-Kindergarten-8th grade, as well as for Algebra, English I, and English II. The overall unit plan is described in the first section of the ELA and math units. This section includes the unit title, a suggested time frame, the grade level MS CCRS addressed and assessed, a unit overview with essential questions and a summary of lesson tasks, and the culminating/performance task description and rubric.

Though the math and ELA overall unit plan designs are very similar, some design aspects differ in order to accommodate the respective requirements of each content area. For mathematics, the first section also provides a segment designated for the Standards for Mathematical Practices (SMPs) addressed in the unit. For ELA, the first section also includes a text set with links to texts (if in the public domain) and a fresh/cold-read task.

The second section of each unit includes lesson plans. Within the lesson plans, provided are lesson-specific MS CCRS, suggested time frames, learning targets, guiding questions, required resources and materials, vocabulary terms and instructional strategies, teacher directions, instructional supports for students, enrichment activities, student handouts, assessments (formative, summative, pre-, and self-), and additional resources to aid in the implementation of the lessons.

Implementation

The intention of the MS CCRS Exemplar Units for ELA and mathematics is to provide educators with resources to understand and implement the MS CCRS effectively. The implementation of the MS CCRS Exemplar Units for ELA and mathematics is voluntary. Additionally, the MDE will provide ongoing support for implementation of the MS CCRS Exemplar Units with initial regional trainings followed by site-specific support through our regional service delivery model. For regional and site-specific training, please contact the MDE Office of Professional Development.

Grade Level	Unit Title Duration			
8 th or 9 th	Understanding Quadratic Functions		8-10 Days	
Mississippi College- and Career-Readiness Standards for Mathematics Standards for Mathem		Standards for Mathematical	Practice	
features of the g technology for n F-IF.7a: Gr	actions expressed symbolically and show key raph, by hand in simple cases and using hore complicated cases. * raph functions (linear and quadratic) and show , maxima, and minima.	SM SMI	 P.1: Make sense of problems and persevere P.2: Reason abstractly and quantitatively. P.3: Construct viable arguments and critique others. P.4: Model with mathematics. 	
factorizations ar	y zeros of polynomials when suitable e available, and use the zeros to construct a he function defined by the polynomial (limit to e polynomials).	SM SM	 P.5: Use appropriate tools strategically. P.6: Attend to precision. P.7: Look and make use of structure. 	
domain) to anot element of the d a function and x the output of f c the graph of the	nd that a function from one set (called the her set (called the range) assigns to each lomain exactly one element of the range. If f is is an element of its domain, then $f(x)$ denotes orresponding to the input x. The graph of f is equation $y = f(x)$.	SM	P.8: Look for and express regularity in repea	ated reasoning.
of the quantities	oret key features of graphs and tables in terms , and sketch graphs showing key features given ion of the relationship. Key features include:			

intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *

F-IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Unit Overview

This unit will focus on analyzing quadratic functions. Topics such as graphing, interpreting, modeling, and solving quadratic equations by factoring and with the use of technology will be explored. Critical thinking questions and discovering multiple methods for analyzing a quadratic function in a real-world context will be a primary focus.

Essential Questions:

- What characteristics and features of a quadratic function are revealed given its initial form?
- What are the advantages of knowing a variety of ways to analyze a quadratic function?
- What information do the key features of a parabolic graph provide?
- Can you use the words roots, solutions, zeros, x-values, and x-intercepts interchangeably? Justify your response algebraically, graphically, and/or technologically.

Lesson Tasks

Lesson 1: Introducing Quadratic Functions

Students will participate in a quick review cycle on the concepts associated with a linear function. They will also identify the key features of the parent function of a quadratic function through technology-discovery activities, geometry, algebraic thinking, and the use of manipulatives.

Lesson 2: Exploring Quadratic Functions Beyond the Parent Function

Students will identify characteristics of quadratic functions written in standard form through the analysis of graphical representations.

Lesson 3: Linear Factors and Standard Form

Students will use the Zero-Product Property (Zero Factor Property) to identify the zeros of polynomial functions in the form y = (ax + b)(cx + d) and $y = ax^2 + bx + c$, and use the zeros to construct a rough draft of the function.

Lesson 4: The Graph Tells It All

Students will combine all the skills they learned over the past few days to write the equation for a quadratic function that has been graphed in the Coordinate Plane.

Lesson 5: Will He Make the Basket?

Students will examine real-world situations and the parabolic graphs that represent them and identify their defining features in real-world context.

Lesson 6: Performance Task—Start Your Engines!

Students will work independently on a performance task containing 2 real-world situations that can be represented by quadratic functions, interpreting the features of the graph in context and comparing the two situations. Student work will be graded by the teacher using a rubric.

Performance/Culminating Task

Start Your Engines!

In this performance task, students will examine the results of a fictional science class's data for an experiment in which they observed toy rockets launched with different accelerants. Given the requirements for the rocket test and the group's equation for the rocket's height after a certain amount of time, students will calculate the vertex, x-intercept, and y-intercept. Using this data, the students will score the science groups using a provided rubric. Students will sketch a graph for each group to compare the two and decide which group was most successful.

Standard Assessed: F-IF.7a

Rubric for Performance/Culminating Task

Level	Mastery Level	Finds Group A's vertex, x-intercepts, and y-intercepts	Reports Group A's scores and explains evidence	Finds Group B's vertex, x-intercepts, and y-intercept	Reports Group B's scores and explains evidence	Sketches and labels both groups' graphs correctly.
4	Exemplifying Mastery	Finds all 4 key features correctly	Reports all 4 scores correctly with evidence.	Finds all 4 key features correctly	Reports all 4 scores correctly with evidence.	Sketches graphs of both groups and labels both correctly.
3	Approaching Mastery	Finds 3 of the 4 key features correctly.	Reports 3 scores correctly with evidence.	Finds 3 of the 4 key features correctly.	Reports 3 scores correctly with evidence.	Sketches and labels both graphs but has one error.
2	Developing Mastery	Finds 2 of the 4 key features correctly.	Reports 2 scores correctly with evidence.	Finds 2 of the 4 key features correctly.	Reports 2 scores correctly with evidence.	Sketches and labels both graphs but has two errors.
1	Not Representing Mastery	Finds 1 of the 4 features correctly.	Reports 1 score correctly with evidence.	Finds 1 of the 4 features correctly.	Reports 1 score correctly with evidence.	Sketches and labels both graphs but has 3 or more errors.
0	No Evidence of Mastery	Does not find any of the 4 features correctly.	Reports 0 scores correctly with evidence.	Does not find any of the 4 features correctly.	Reports 0 scores correctly with evidence.	Sketches are missing or show more than 3 errors.

Lesson 1: Introducing Quadratic Functions

Focus Standard: F-IF.7a

Additional Standards: F-IF.1, F-IF.4

Standards for Mathematical Practice: SMP.1, SMP.2, SMP.3, SMP.4, SMP.5, SMP. 6, SMP.7, SMP.8

Estimated Time: 60 minutes – 180 minutes

Resources and Materials:

- Bell or grade appropriate music
- Cardstock
- Colored pencils or highlighters
- Document Camera (optional)
- Glue
- Laminating machine or thick, clear packing tape
- Markers
- Miras (optional)
- Notebook Paper
- Questions 4 Quadratics (Q4Q) and Answers 4 Quadratics (A4Q) Wall
 Note: You can "create" a wall or section of a part of your Word Wall. Use masking tape as a divider.
- Ruler (optional)
- Scissors
- TI-83/TI-84
- Wall Tape
- Handout 1.1: Give Me Five
- Handout 1.2: One-Tab Notebook Foldables[®] Template (6 copies per student)
- Handout 1.3: Calculator Help Table Tents (optional)
- Handout 1.4: Text Message Summary Conversation
- Handout 1.5: I See Parabolas in the World Around Me (optional)
- Desmos Graphing Calculator: <u>www.Desmos.com</u>

• MAAP Item Sampler: <u>https://ms-sampler.nextera.questarai.com/tds/#practice</u>

Lesson Targets:

- Students will review key vocabulary concepts associated with linear functions.
- Students will compare and contrast linear functions and quadratic functions.
- Students will be introduced to the parent function of a quadratic function and the function $y = -x^2$.
- Students will conceptually understand the key features of the parent function for a quadratic function.

Guiding Questions:

- How are a linear function and a quadratic function similar/different?
- How are the graphs of a linear function and a quadratic function similar/different?
- What connections can be made between the key features of a quadratic function?
- What are some real-world examples of a quadratic function (i.e. parabolic in shape)?

Vocabulary		
Academic Vocabulary: Axis of Symmetry Coordinate Point(s) Decreasing Domain End Behavior Equidistant Function Increasing Linear Function Maximum Midpoint Minimum Parabola	 Instructional Strategies for Academic Vocabulary: Introduce academic vocabulary with student-friendly definitions and pictures Model how to use academic vocabulary in discussion Discuss the meaning of an academic vocabulary word in a mathematical context Justify responses and critique the reasoning of others algebraically, geometrically, and/or technologically using academic vocabulary Create pictures/symbols to represent academic vocabulary Write or use literacy strategies involving academic vocabulary 	

 Parent Function Quadratic Function Range Rate of change Satisfies Solution Standard Form Vertex x-intercept y-intercept 		
Symbol	Interpretation of Symbol	
	Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level	
\checkmark	Assessment (Pre-assessment, Formative, Self, or Summative)	

Instructional Plan

Understanding Lesson Purpose and Student Outcomes: Students will be able to effectively communicate about the key features of a quadratic function with and without technology.

Anticipatory Set/Introduction to the Lesson: Give Me Five

Note: Before class begins, arrange the classroom so that student desks are grouped in pairs. Copy **Handout 1.1: Give Me Five** onto white cardstock, laminate each page, and cut out each item. If a laminating machine is not available, clear packing tape will work as well.

Tape one set of hands found on **Handout 1.1: Give Me Five** approximately 2 feet apart on the wall. One vocabulary card and one function card will be placed between each set of hands, with the vocabulary card on top. This will create "stations" on the walls around the classroom.

Form student pairs where one student is above/on grade level and the other student is below/on grade level based on previous data collected. Student pairs should be posted on the wall or displayed on the interactive whiteboard to facilitate grouping and initiation of the activity.

During this activity, instruct students to place either their left hand or right hand on one of the hand templates. Each student must say the vocabulary word at their "station" aloud and provide a student-friendly definition to their partner. Using their definition, the student must discuss each vocabulary word and identify the relationship to the function underneath. Each student is given 30 seconds to complete this task at each "station." Prior to rotating students to the next "station," instruct students to give their partner a "high five" to provide positive reinforcement. By making students place their hands on the hand templates, students are focused on the task and allocate an equal amount of time and attention to each "station."

Rotate students counterclockwise around the room through the use of a bell or grade-appropriate music until all students return to their original "station." Circulate between groups prompting students with clarifying questions (SMP.1-8). For example:

- ✓ How does this vocabulary word/graph differ from the vocabulary word/graph at the previous "station?"
- ✓ Is there another word that is synonymous with _____? Do these two words mean the same thing all the time?
- ✓ Is there one thing your partner said that you may disagree with? If so, explain your reasoning.
- ✓ What key feature about this graph provides the evidence that the ______ is _____?
- ✓ What role does the graph play in helping you to relate the vocabulary word ______with the graph shown?

At the completion of this activity, have students return to their desks. Select the "stations" that have the three quadratic functions and highlight some of the discussions that you heard while circulating throughout the groups.

Note: (1) This activity is designed for a maximum class size of 30. For smaller classes, students may work independently and place both of their hands on the hand templates. (2) If the latter is the case, you may circulate throughout the room, place your hands on one of the templates, and serve as a partner for any student that may be struggling. (3) The two function cards that have a symbol represent a visual cue to be mindful about which vocabulary cards you pair with them.

For students who are EL, have disabilities, or perform well below grade level:

• Provide the vocabulary terms from **Handout 1.1: Give Me Five** to the class the day before the lesson and allow them to review the terms overnight in preparation for the lesson.

Extensions for students with high interest or working above grade level:

• Provide the graph of the quadratic functions from **Handout 1.1: Give Me Five** to the class the day before the lesson and ask them to write down one question they have about each graph. Selected questions may be collected and used as exit tickets at the end of the lesson, discussed with the entire class where appropriate in the lesson, or they may be posted on the "Questions for Quadratics" (Q4Q) Wall.

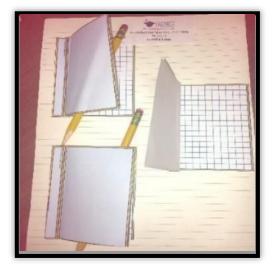
Activity 1: Understanding the Key Features of the Parent Function $y = x^2$

Quickly display each vocabulary word from the Give Me Five activity and do a quick Math Talk with students to ensure they have a clear understanding of each vocabulary word and how it relates to each *linear function* it is paired with. Pose the following questions to introduce students to quadratic functions. As students respond, place each question and correct student response(s) on the "Questions for Quadratics" and "Answers for Quadratics" Wall (Q4Q and A4Q Wall).

- ✓ What observations did you make about the 3 graphs that were shaped like the letter "U?"
- ✓ What do these "U" shaped graphs remind you of?
- ✓ Provide one characteristic that these 3 graphs have in common with each other.
- ✓ Can you list two differences that exist between the "U" shaped graphs and the graphs of the linear functions?

Distribute six copies of **Handout 1.2: One-Tab Notebook Foldables**[®] **Template** to each student. This handout will serve as their note- taking document for this activity and Activity 1 on the following days. Instruct students to cut along the perforated lines and between each tab creating a series of tabs. They will then fold the anchor tab for the graphing page and glue it onto a sheet of notebook paper. Students then fold the anchor tab.

Note: (1) If time or resources are a concern, pre-cut each foldable prior to class and distribute them to each student or request students use notebook paper to create a similar foldable. (2) The graphs used for this activity do not have the axes labeled. This will allow students to attend to precision and scaling as they create their own graphs (SMP.6).



For students who are EL, have disabilities, or perform well below grade level:

• Provide coordinate planes where both, the gridlines and axes, are pre-labeled.

Progress by explaining that the U-shaped graphs located at a few of the stations are called *parabolas* and that they are the result of graphing a second-degree polynomial called a *quadratic*. Display/write the function rule $y = x^2$ (or use function notation, $f(x) = x^2$) on the Interactive whiteboard and label this as the "Parent Function" of a quadratic function. Explain that this function is written in standard form $y = ax^2 + bx + c$. Ask one student to identify the value of *a*, *b*, and *c* for the parent function. Activate prior knowledge by asking one student, "What is a Parent Function?" based on previous lessons on linear functions. Instruct students to copy this information on the top of tab #1.

Using the 2^{nd} GRAPH (table) function on their calculator, students will copy the *x*-*y* table for the domain *x* = {-5, -4, -3, -2, -1, 0, 1, 2, .3, 4, 5}. To continue activating prior knowledge, instruct students to label the top of their *x*-*y* table with the vocabulary words "Domain" and "Range."

(partial table)		
Domain	Range	
(x)	(y)	
-4	16	
-3	9	
-2	4	
5	25	

✓ Using a few entries from their *x*-*y* table, ask students to make a very quick observation about the rate of change for the parent function $y = x^2$ and contrast this with the rate of change for a linear function.

Note: Average of rate of change is not needed at this point in the lesson.

✓ Prove that any point (*x*, *y*) algebraically satisfies the function $y = x^2$.

Example: (-5, 25)		Example: (-3,9)
$y = x^2$	Function	$y = x^2$
$25 = (-5)^2$		9 = (-3) ²
25 = 25 🗸		9 = 9 🗸

Encourage students to come to a consensus as a class about how they will label their axes prior to sketching the graph of the function on their Coordinate Plane tab (SMP. 6). Instruct students to do a Pairs Check and turn to their partner to verify that they have created the same graph.

Pose the following seven questions to students. Allow ample wait time between questions. Ensure that all students are competent in using the 2^{nd} TRACE function on the calculator to respond to question #6. Then select various students to share their response(s) with the entire class, and where necessary, allow them to come to the Interactive whiteboard to justify their response.

- ✓ What can you assume about the end behavior of your graph? How do you know this?
- ✓ Identify any other coordinate points that may lie on the graph aside from the ones listed in your table.
- ✓ What can you tell me about the graph after you plot the coordinate points on your graph?
- ✓ Why is every coordinate point that lies on the graph considered a "solution?"
- ✓ Is there a point where the graph begins to turn and go in the other direction? How can you find the exact coordinates of this point using technology?
- ✓ Is there a way to algebraically, graphically, and/or technologically verify the x-intercept and y-intercept of the parent function?
- ✓ Describe the interval on which the graph appears to be increasing/decreasing.

Students will continue to use the top of tab #1 or the margins of their notebook to paraphrase the following notes in their own words:

• The term "quadratic" has Latin origin and derives from the word "quadratum" or "quadrus" which means square.

Note: Ask students can they find the first few letters of quadrus in the word square and circle them.

- The standard form of a quadratic function is $y = ax^2 + bx + c$, where in our first example, "a" = 1, "b" = 0 and "c" = 0; and note that "x" and "y" can always be replaced with other variables as seen in previous lessons on linear functions.
- Quadratic functions have some graphical key features that most linear functions have, namely an *x*-intercept (or zero), *y*-intercept, domain, and range.

• Recall that you can algebraically and technologically prove that the x-intercept is the coordinate point where y = 0 and the y-intercept is the coordinate point where x = 0.

Note: Students should be familiar with using the TI-83/TI-84 calculator to identify these graphical features (i.e. 2^{nd} TRACE). Encourage them to do so and model for those that may have forgotten.

<i>x</i> -intercept		y-intercept
$y = x^2$	Function	$y = x^2$
$0 = x^2$		$y = O^2$
$\sqrt{0} = \sqrt{x^2}$		y = 0
0 = <i>x</i>		
(0,0)	Coordinate Point	(0,0)

For students who are EL, have disabilities, or perform well below grade level:

- Provide **Handout 1.3: Calculator Help Table Tents**. Have the student cut along the outside edges, fold along the solid black line, and stand it up on their desk to use as a reference throughout the unit.
- Quadratic functions have some graphical key features that linear functions don't have, namely a vertex, an axis of symmetry, and intervals where the function increases and decreases.

Note: At this point, ask a student if they can identify these intervals for the parent function.

- The "vertex" is the coordinate point where the graph begins to change direction. It is also the minimum of our parent function.
- Because the vertex is the point where the graph begins to turn and go in the opposite direction, it provides key information about our range.

Note: At this point, ask a student if they can identify the range for the parent function.

• The TI-83/TI-84 calculator can be used to identify the coordinates of the vertex with accuracy.

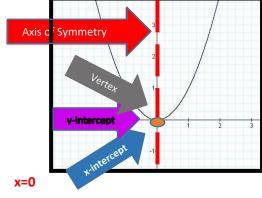
For students who are EL, have disabilities, or perform well below grade level:

- Provide **Handout 1.3: Calculator Help Table Tents**. Have the student cut along the outside edges, fold along the solid black line, and stand it up on their desk to use as a reference throughout the unit.
- Directly through the center of the graph, at the vertex, is an imaginary vertical line known as the "axis of symmetry." This line divides the graph into two equal parts, basically acting like a mirror.

For students who are EL, have disabilities, or perform well below grade level:

- Allow only students that need a visual to quickly use a Mira[®] to see *how* the axis of symmetry acts like a mirror dividing the quadratic function into two equal parts.
- Because the axis of symmetry is an imaginary line, it is it is usually represented with a dotted/dashed line.
- The point where the graph intersects the axis of symmetry is the vertex.

Model and instruct students to identify these key features on their Foldable using different colored pencils to distinguish the key features of the parent function.



Circulate throughout the room verifying student work and ask students the six questions listed below. Allow ample wait time between questions and encourage students to provide an explanation to their partner (SMP.3). Select various students to share their response(s) with the entire class.

Note: The first five questions should be asked in some format during the entire unit.

- 1. Will the x-intercept, y-intercept, and vertex always be at the same location/coordinate point?
- 2. What can you assume is the equation for the axis of symmetry?
- 3. Revisit the leading coefficient of our function. What role do you think the leading coefficient has on the direction and shape of the graph?
- 4. Revisit the leading coefficient of our function. What role do you think the leading coefficient has on the location of the vertex?
- 5. Revisit the constant term of our function. What role do you think the constant term has on the graph of our quadratic?
- 6. Do you think the answers to these questions will be the same for all values of a, b, and c?

For students who are EL, have disabilities, or perform well below grade level:

- Give the midpoint (*m*, *n*) on the horizontal line segment connecting the points (*x*, *y*) and (-*x*, *y*) that lies on the axis of symmetry. Then, allow them to count the number of "squares" on the coordinate plane between the point (*m*, *n*) and the point (*x*, *y*) to approximate the distance. Repeat this process to approximate the distance between the points (*m*, *n*,) and (-*x*, *y*).
- An alternative process would be to allow them to use a ruler to measure the distance between the point (*m*, *n*) and the point (*x*, *y*). Repeat this process to measure the distance between the points (*m*, *n*) and (-*x*, *y*).

Extensions for students with high interest or working above grade level:

- Allow students to select any point (*m*, *n*) on the axis of symmetry and any two points (*x*, *y*) and (-*x*, *y*). Instruct them to use the distance formula to verify equidistance.
- Allow students to use the midpoint formula to verify that the coordinate point (*m*, *n*) which they have selected on the axis of symmetry is halfway between the two points (*x*, *y*) and (-*x*, *y*).

Activity 2: Evaluating Quadratic Functions in the Form $y = -x^2$

Display the function rule $y = -x^2$ (or use function notation, $f(x) = -x^2$) on the Interactive whiteboard. Instruct students to write this function on the top of tab #2 and to create a t-table using their calculator's Table function for the same domain in the previous example,

 $x = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$. Students will proceed to graph the function $y = -x^2$ on their coordinate plane tab and identify the graph's key features as previously done. Circulate throughout the room and verify student work and calculator usage.

Ask students to talk to their partner about the similarities and differences that exist between the parent function $y = x^2$ and the function $y = -x^2$.

Note: Take this opportunity to revisit the aforementioned six questions and pay close attention to student responses for questions #3 and #4 as they use their new academic vocabulary (SMP.3 and SMP.6).

Reflection and Closure: Text Message Summary

Instruct students to separate their desks so they can complete this activity independently. Distribute **Handout 1.4: Text Message Summary Conversation** to each student. Tell them to pretend that their best friend was absent during today's lesson. Explain that their task is to construct a text message conversation summarizing the lesson and what they learned today. Encourage students to:

- keep their conversation within each conversation bubble.
- use their colored pencils to create emoji.
- use at least 3 academic vocabulary words.

Select a few students to share/read their work with the class.

Note: If a document camera is available, it may be used to facilitate viewing each student's work sample. Observe which vocabulary words are/aren't used during the student presentations to determine if additional supports are needed.

Homework

Students will participate in a scavenger hunt to snap pictures on their cell phone of real-world objects that resemble the graph of a quadratic function. If a student does not have a cell phone, they may use any form of printed material (magazine, newspaper, images from the internet) to complete this homework activity. Award points based on the number of occurrences a student is able to find. How the points will be used is the teacher's discretion (e.g. extra credit, points on the next test, a free homework pass, etc.).

Note: Be sure to get prior approval from your administrator to allow students to use their cell phones in the classroom for this activity. If your school policy prohibits the use of cell phones in the classroom, instruct students to email the photos to your designated school email address and print them out.

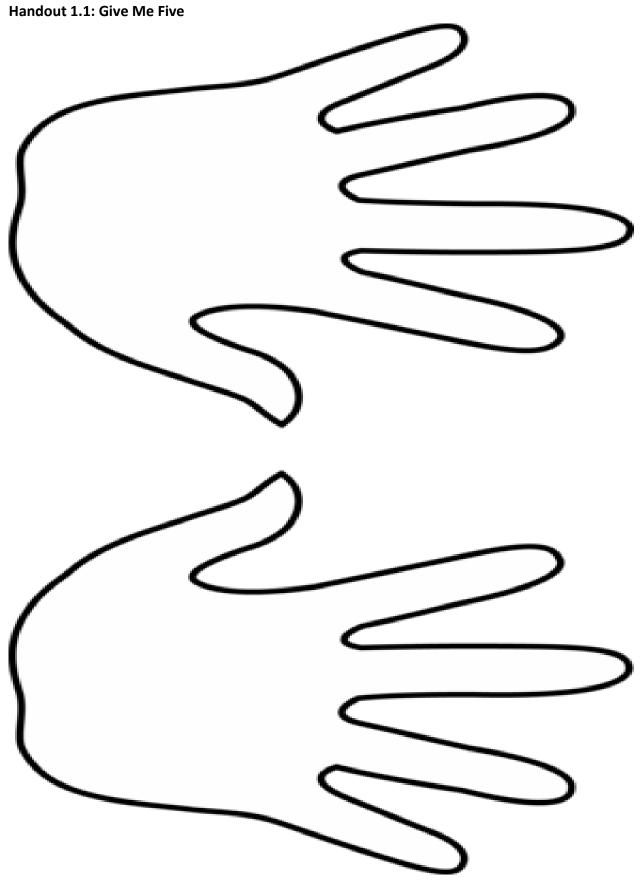
For students who are EL, have disabilities, or perform well below grade level:

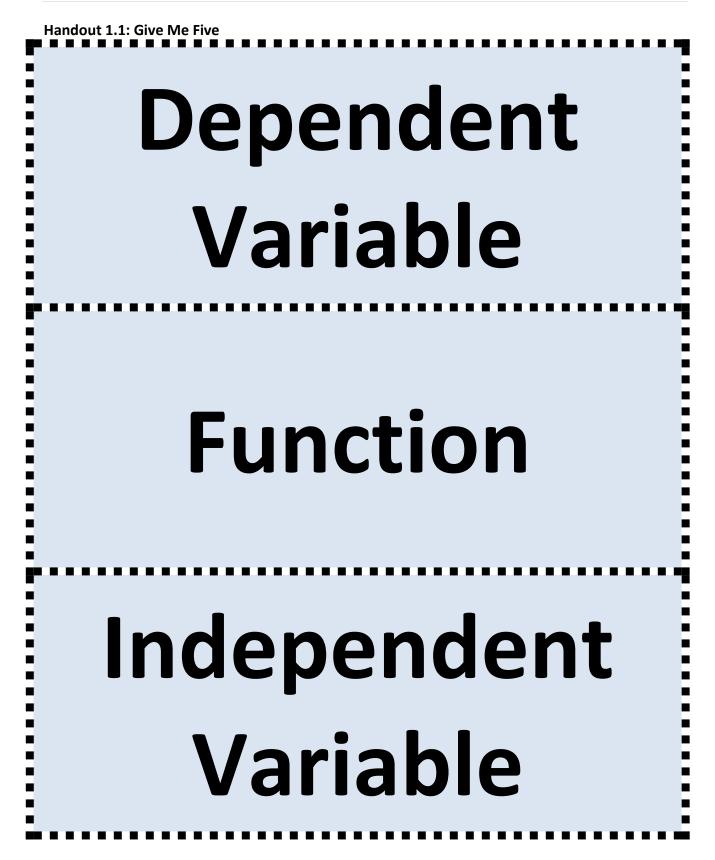
• Provide **Handout 1.5: I See Parabolas in the World Around Me** and instruct them to place a check mark in the box for the objects that are parabolic in shape. You may replace some of the photos with objects in the city, neighborhood, or classroom.

Note: Ensure that you include enough photos that are parabolic in shape to give the student the opportunity get the maximum number of points available.

• For EL students, you can use the internet and replace some of the photos in the document with objects from their native country.

Extra Credit: Assign item number 46 from the <u>MAAP/Questar Practice Test</u>. Require a detailed explanation for responses.

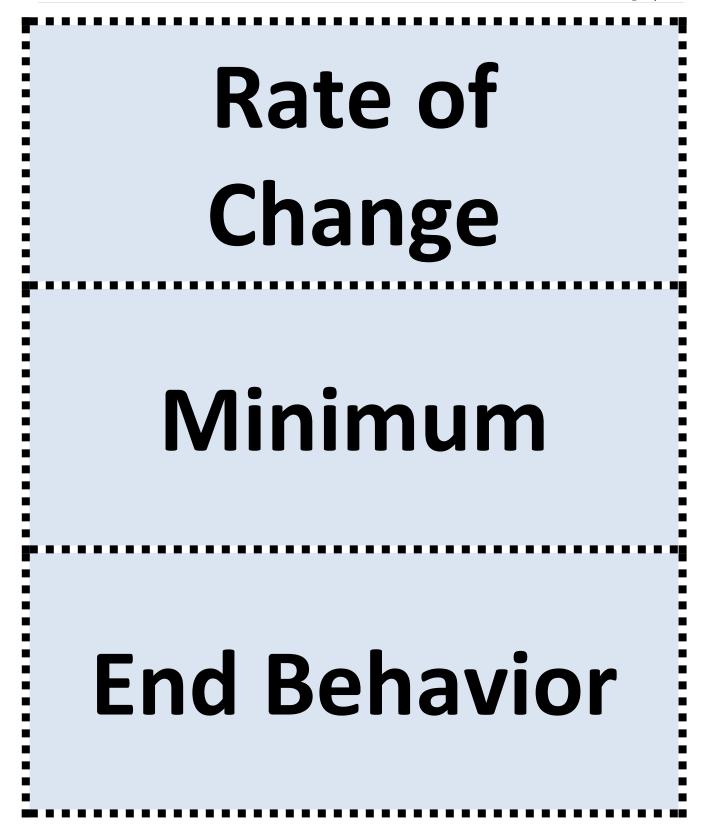


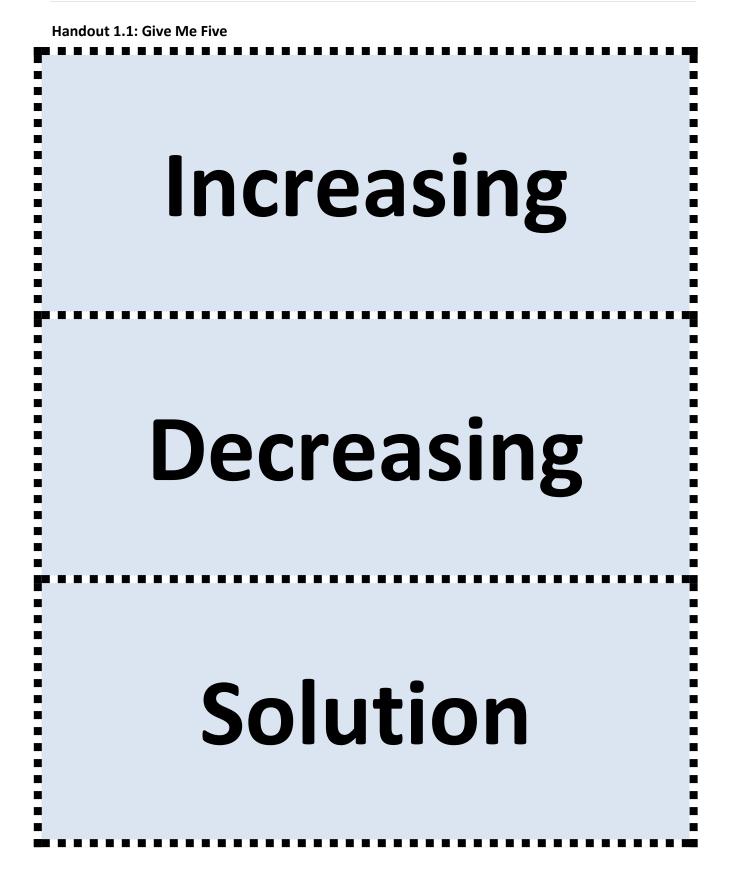


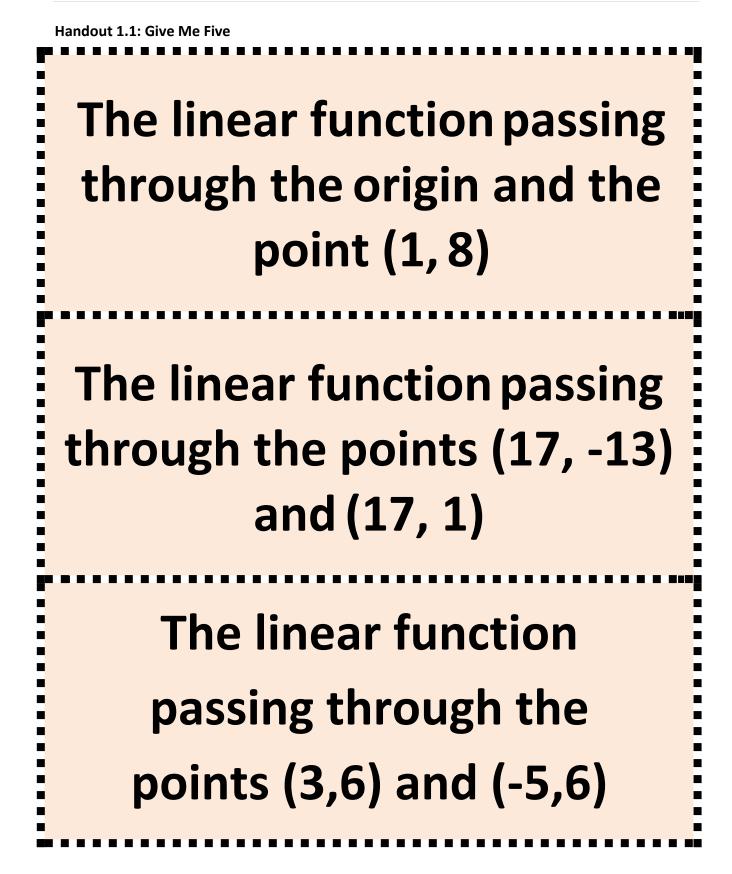
Handout 1.1: Give Me Five x-intercept y-intercept Domain

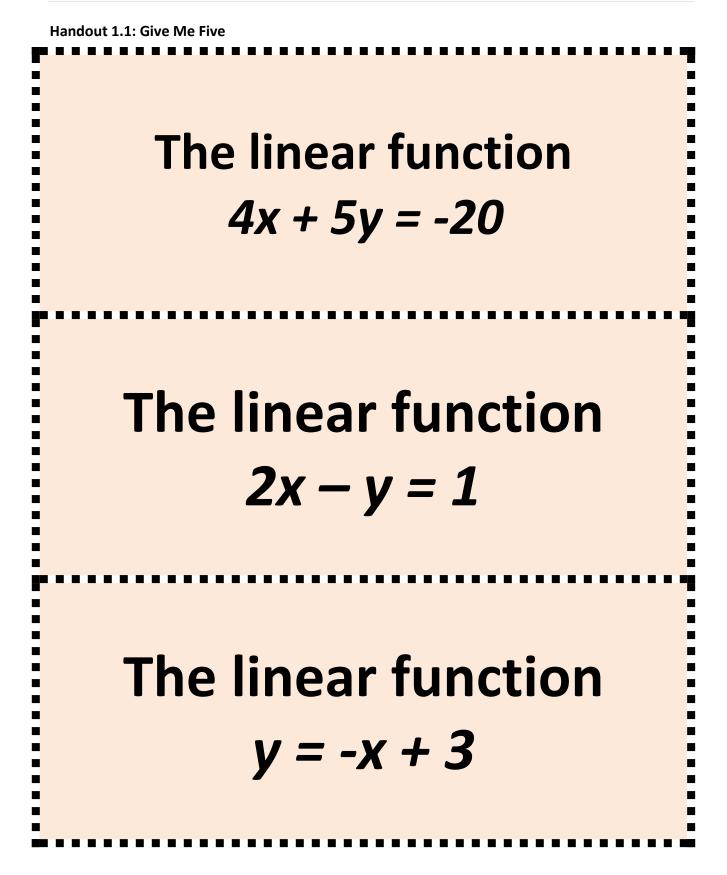
Handout 1.1: Give Me Five

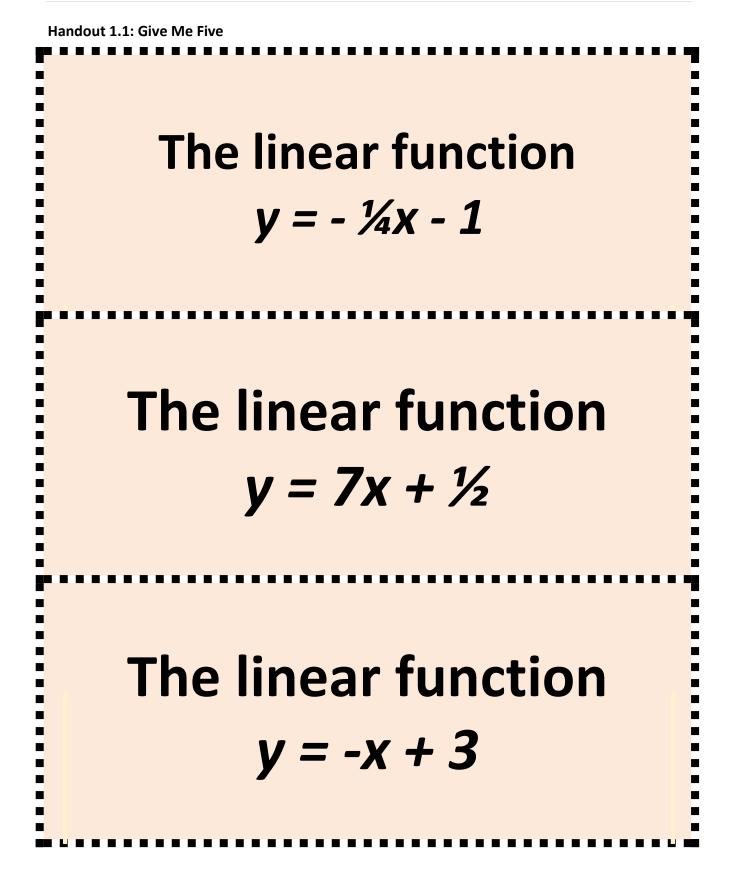
Range
Standard Form
Maximum

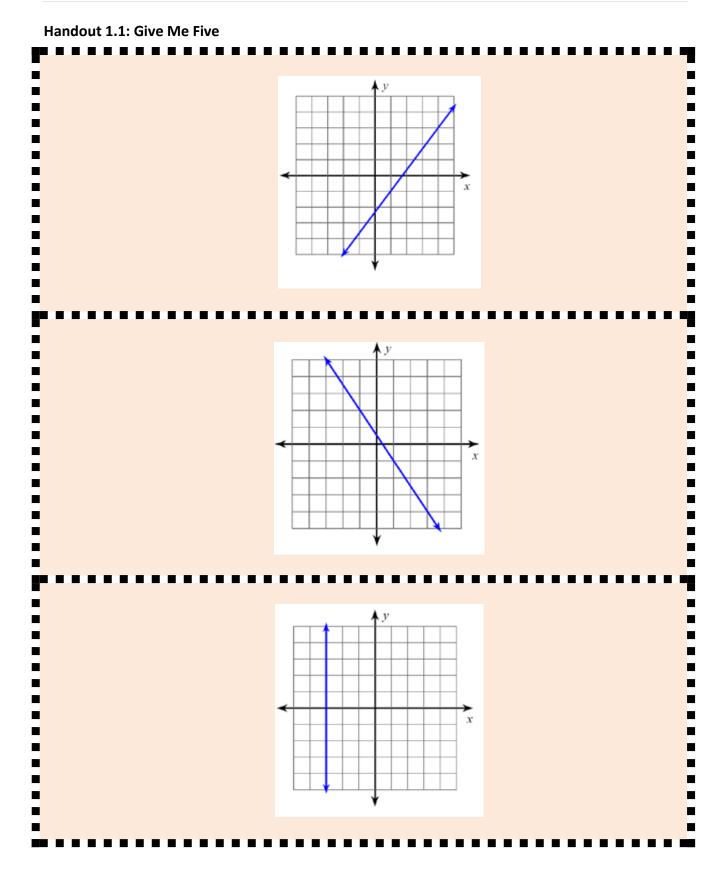


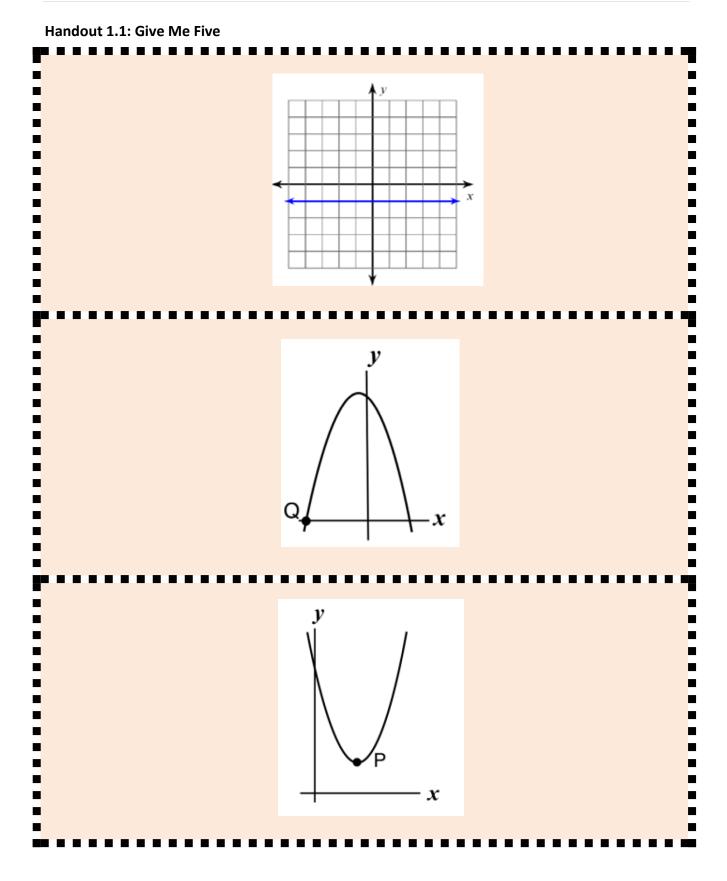


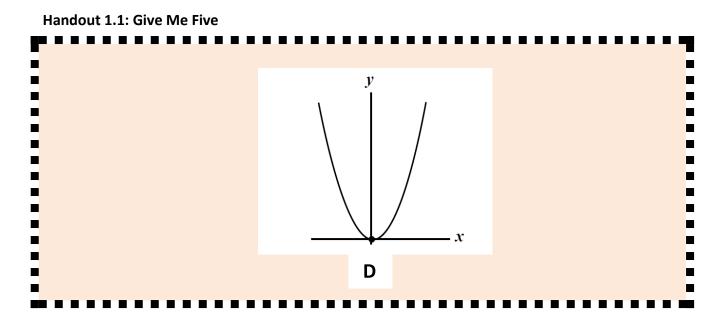




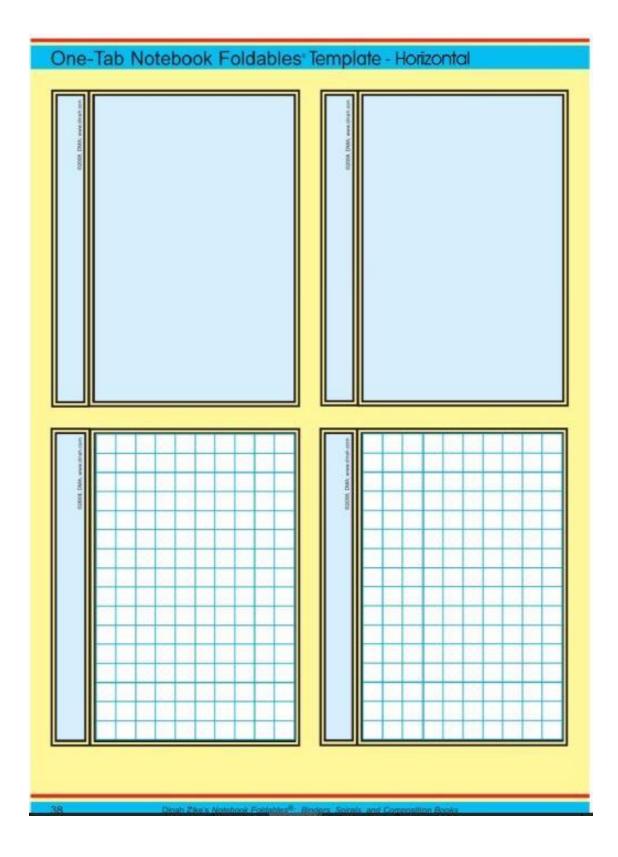




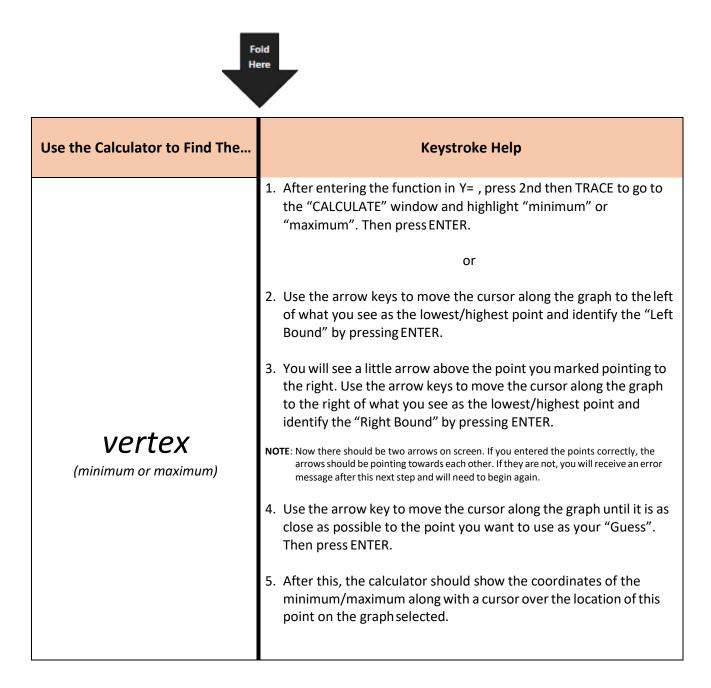




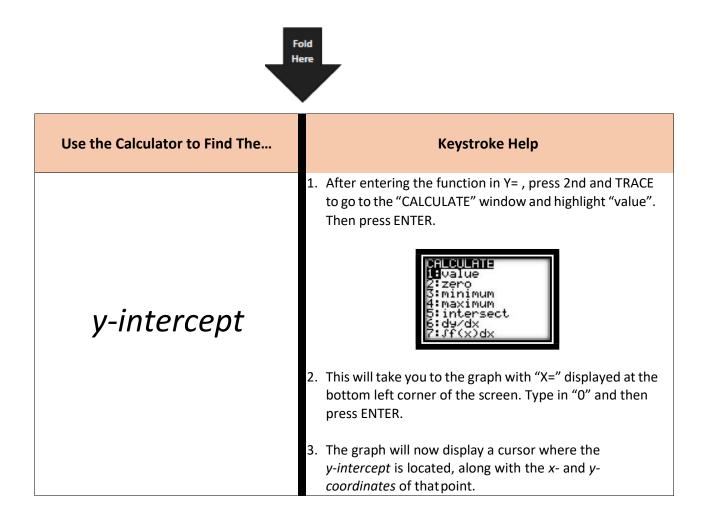
Handout 1.2: One-Tab Notebook Foldables® Template

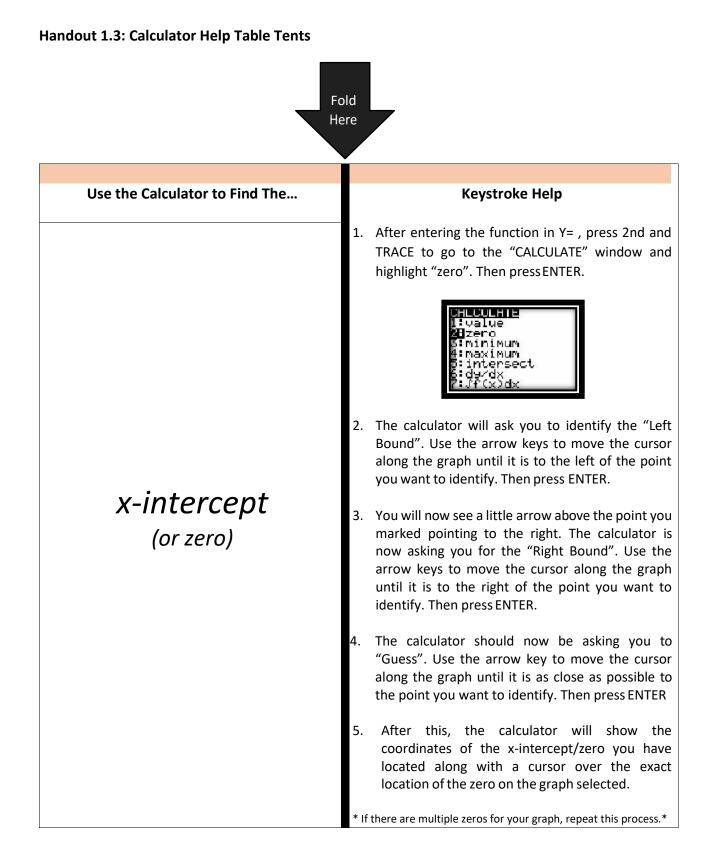












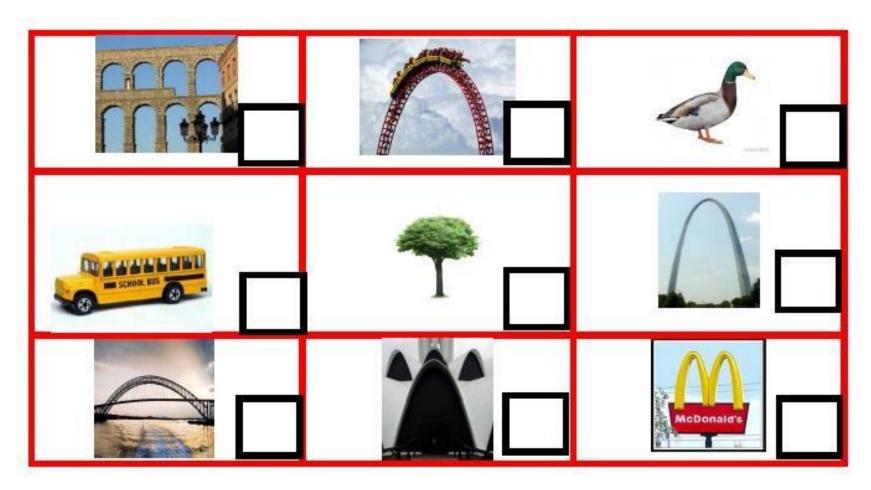


Handout 1.4: Text Message Summary Conversation

Handout 1.5: I See Parabolas in the World Around Me

Name:_____

Date: _____



Lesson 2: Exploring Quadratics Beyond the Parent Function

Focus Standard: F-IF.7a

Additional Standard: F-IF.1, F-IF.4

Standards for Mathematical Practice: SMP.1, SMP.2, SMP.3, SMP.4, SMP.5, SMP. 6, SMP.7, SMP.8

Estimated Time: 60 minutes – 180 minutes

Resources and Materials:

- Cardstock
- Chart paper (with an adhesive back)
- Colored pencils or highlighters
- Document Camera (optional)
- Domino
- Markers
- Notebook Paper
- Post-it Notes (lined)
- Questions 4 Quadratics (Q4Q) and Answers 4 Quadratics (A4Q) Wall
- TI-83/TI-84
- Handout 2.1: Say it with a Venn
- Handout 1.2: One-Tab Notebook Foldables® Template (from the previous day's lesson)
- Handout 2.2: Graphing Quadratic Functions Where b≠ 0
- Handout 2.3: The Conjecture's All Mine
- Desmos Graphing Calculator: <u>www.Desmos.com</u>
- Exploring Quadratic Functions: <u>https://www.geogebra.org/m/RVF4GYcX</u> (optional)

Lesson Target:

• Students will be able to effectively communicate in a variety of ways about quadratic functions in the form $y = ax^2 + bx + c$, where *a*, *b*, and *c* are Real Numbers.

	Vocabulary
Academic Vocabulary: Axis of Symmetry Coordinate Point(s) Decreasing Domain Increasing Maximum Midpoint Minimum Parabola Parent Function Quadratic Function Range Satisfies Solution Standard Form Vertex x-intercept y-intercept	Instructional Strategies for Academic Vocabulary: Introduce academic vocabulary with student-friendly definitions and pictures Model how to use academic vocabulary in discussion Discuss the meaning of an academic vocabulary word in a mathematical context Justify responses and critique the reasoning of others algebraically, geometrically, and/or technologically using academic vocabulary Create pictures/symbols to represent academic vocabular Write or use literacy strategies involving academic vocabulary vocabulary

Symbol	Type of Text and Interpretation of Symbol
	Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level
✓	Assessment (Pre-assessment, Formative, Self, or Summative)
	Instructional Plan

Understanding Lesson Purpose and Student Outcomes: Students will competently evaluate the function form and graphical form of a quadratic function, $y = ax^2 + bx + c$, where *a*, *b*, *and c* are Real Numbers.

Anticipatory Set/Introduction to the Lesson: Say it With a Venn

Have students form eight small groups by using the technique Numbered Heads (i.e. count off by ones up to the number 8). Direct all the ones to a location along the wall, direct all twos to a location along the wall, direct all threes to a location along the wall, and so forth. Distribute a few markers and a sheet of chart paper (with an adhesive back) to each group. Instruct everyone in the group to point to someone else in the group on the count of three. Then count aloud "1,2,3." The student in each group with the most number of fingers pointing at them is the designated "scribe." The scribe will be responsible for capturing the group's responses for this activity.

Display **Handout 2.1: Say It with a Venn** using the Interactive whiteboard or document camera. Read the directions aloud and ask if everyone understands the directives. Circulate throughout the groups and invite students to justify their thinking to their group prior to it being recorded by the scribe (SMP.1-7).

Note: (1) Encourage students to use their academic vocabulary. (2) Encourage the scribe to sketch each graph as the label for the three circles on their Venn Diagram. (3) Consider having a separate, hard copy in your hand as you circulate throughout each group. This will facilitate immediate teacher-to-student discussions.

For students who are EL, have disabilities, or perform well below grade level:

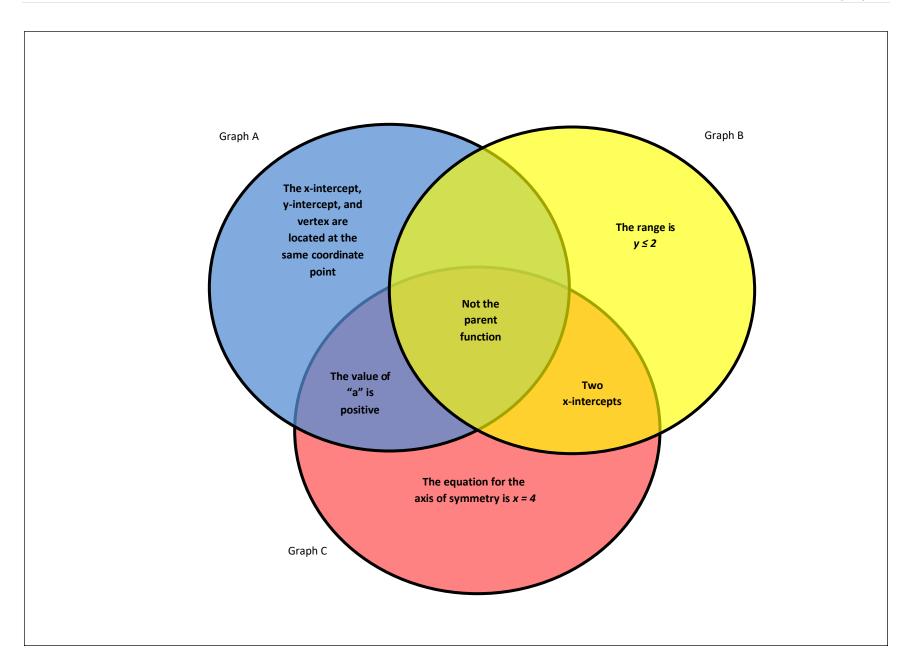
• Have the student point to key features of the graph as s/he provides response(s).

Extensions for students with high interest or working above grade level:

- Verbally, provide the student with some prompting questions for the group, such as:
 - Is this the graph of the parent function? Justify your response.
 - What is unique about this graph?
 - What can we assume about the end behavior of this graph to help us out?
 - What is the relationship between the location of the vertex and the value of "a?"
 - Is there something we can put in all seven regions of the Venn Diagram?

Ask each group to stand in a straight line, by height, in front of their chart paper. The student with the median height must remain at the chart paper and serve as the "reporter." Instruct the remaining students to participate in a very quick Gallery Walk to view the work of the other groups, prior to returning to their seats.

Allow the "reporter" to discuss their group's responses with the entire class. Encourage students from the other groups to validate or respectfully dispute any response they hear (SMP.3). Be prepared to provide a suggestion for those regions that students may have found challenging to write something in.



Activity 1: Learning More about the Impact of the Leading Coefficient, "a"

Instruct students to retrieve their notebook foldable from the previous day and explain that they will continue to use the remaining tabs to take notes and graph (Tabs #3-12).

Display the function rule $y = 2x^2$ (or use *function notation*) on the Interactive whiteboard or whiteboard. Instruct students to write this function on the top of tab #3 and to create a t-table using the same domain from the previous day's example, $x = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$. Students should proceed to graph the function $y = 2x^2$ on their graphing tab and identify the graph's features.

Note: Encourage students to use their graphing calculator (i.e. 2nd TRACE) to verify the graph's features. Remind those students that received a "Calculator Help Table Tent" on the previous class day that they may reference that as they continue to work (SMP.4).

Conduct a quick Math Talk using the first five questions given in the previous day's lesson. Draw emphasis on the second half of question numbers three and four. Be sure to add/put student responses on the A4Q Wall.

- Will the x-intercept, y-intercept, and vertex always be at the same location/coordinate point?
- What can you assume is the equation for the axis of symmetry?
- Revisit the leading coefficient of our function. What role do you think the leading coefficient has on the direction and shape of the graph?
- Revisit the leading coefficient of our function. What role do you think the leading coefficient has on the location of the vertex?
- Revisit the constant term of our function. What role do you think the constant term has on the graph of our quadratic?

Instruct students to predict what the graph of $y = -2x^2$ would look like. Select one student to come to the front of the room to display the t-table for this function. Ask a volunteer to sketch the graph of the parent function in one color and the graph of $y = -2x^2$ in another color. On tabs #4 - #8, students will complete t-tables for the function rules $y = \frac{1}{2}x^2$, $y = 0.625x^2$, $y = \frac{3}{4}x^2$, $y = -\frac{3}{4}x^2$, and $y = 4x^2$, respectfully. Be sure to come to a consensus as a class how you will label the axis for each function before graphing on the coordinate plane tabs underneath (SMP.6).

Note: Consider not writing one or two of the functions in standard form where the leading coefficient and the variable "x" are on the other side of the equal sign with the variable "y" (e.g. $y - 4x^2 = 0$). Students should be familiar with re-writing functions in the form of "y =" based on previous lessons on linear functions.

Continue to facilitate the discussion. Ask the following questions, or similar questions. Be sure to add/put student responses on the A4Q Wall.

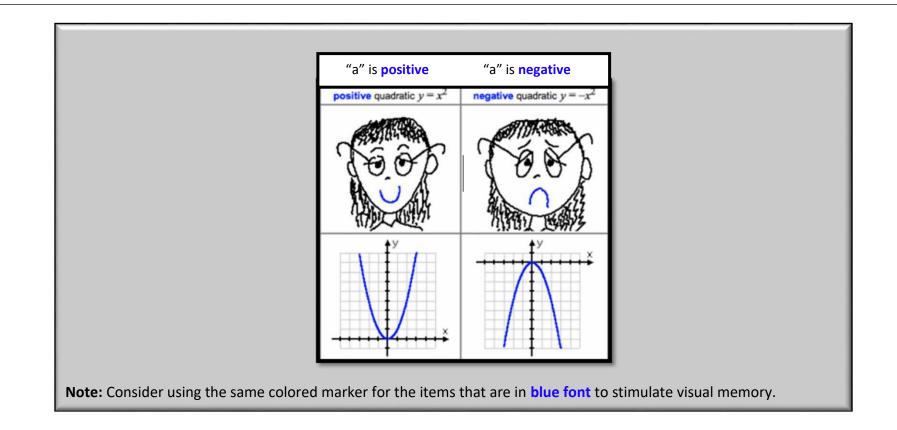
- \checkmark Can we graph a function when the variables "x" and "y" are on the same side of the equal sign? Justify your response.
- \checkmark Can you explain the importance the value of "a" has when graphing quadratic functions?
- ✓ If a function rule has both "x" and "y" on the same side of the equal sign, which variable must we ensure is positive prior to graphing? Justify your response.

Once students are finished, they will complete a quick writing prompt in the right margin or at the bottom of their notebook paper using the following stems. Be sure to add/put student responses on the A4Q Wall.

- ✓ "In comparison to the parent function, y= x², when | a | < 1, the graph______
 This is because______
- ✓ "In comparison to the parent function, y= x², when | a | > 1, the graph______
 This is because______

For students who are EL, have disabilities, or perform well below grade level:

• Remind them that the **value of a** (positive or negative) determines the direction of the graph if the graph opens upward or downward. Provide them with a visual clue before they complete their response to this writing prompt. Encourage them to copy this visual in the margins of their notebook.



When students have completed their response, instruct them to stand up and quickly find a partner across the room and share their responses and discuss. Encourage them to ask each other clarifying questions, if necessary (SMP.3 and SMP.6).

Prior to returning to their seat, invite students to find a different partner and simply ask the following questions without responding:

- "What impact do you think it would have on the parent function, $y = x^2$, if the value of "c" is less than 0? "
- "What impact do you think it would have on the parent function, $y = x^2$, if the value of "c" is greater than 0?"

Students will take a few minutes to brainstorm silently upon returning to their seats.

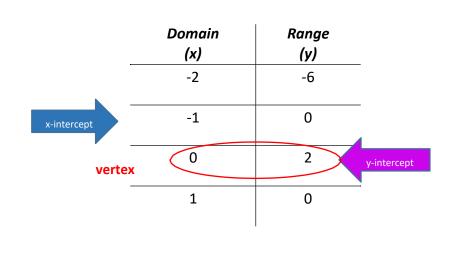
Activity 2: Learning More about the Impact of the Constant Term, "c"

On tabs #9 - #12, students will write the function rules for the following functions and sketch them using the table function on their calculator: $y = 1x^2 + 2$, $y = 2x^2 - 3$, $f(x) = -\frac{1}{2}x^2 + 0.5$, and $y = -2x^2 + 2$.

Note: Consider not writing one or two of the functions in standard form where the leading coefficient, the variable "x", and constant term are all (or some of them are) on the other side of the equal sign with the variable "y" (e.g. $y + 3 = 2x^2$).

Encourage students to use their graphing calculator (i.e. 2^{nd} TRACE) to verify the graph's features. Instruct them to use a highlighter or colored pencil to identify the *x*-intercept, *y*-intercept, and vertex in their table and emphasize these features on each graph.

(Example of the partial table for $y = -2x^2 + 2$)



MS Exemplar Unit
• Mathematics

Select two students	to come to the front of the room and display their work. Conduct a quick Math Talk using the previously stated five
questions. Draw em	phasis on the second half of question number five. Be sure to add/put student responses on the A4Q Wall.
	Will the x-intercept, y-intercept, and vertex always be at the same location/coordinate point?
0	What can you assume is the equation for the axis of symmetry?
0	Revisit the leading coefficient of our function. What role do you think the leading coefficient
	has on the direction and shape of the graph?
0	Revisit the leading coefficient of our function. What role do you think the leading coefficient
	has on the location of the vertex?
0	Revisit the constant term of our function. What role do you think the constant term has on
	the graph of our quadratic?
✓ "In cor	Be sure to add/put student responses on the A4Q Wall. nparison to the parent function, y= x ² , when c < 0, the graph because"
	nparison to the parent function, y= x ² , when c > 0, the graph
	because
1113 13	Decause
Note: Encourage st range in their respo	udents to include details about the location of the vertex with respect to the <i>x-axis,</i> the axis of symmetry, and the inse.

Activity 3: Learning More about the Impact when $b \neq 0$

Select a few of the functions from the notes students have been taking over the past few days (tabs #2-12) and graph them on the same coordinate plane as the parent function, $y = x^2$, one at a time. Demonstrate how to change the graphing style in Y= on the TI-83/TI-84 to distinguish between the two graphs visually (i.e. by making one graph darker than the other)

(SMP.4).

lcon	Style	Description
N	Line	A solid line connects plotted points; this is the default in Connected mode
N.	Thick	A thick solid line connects plotted points

Instruct students to put their Foldable inside their class binder and explain they will use this calculator technique to complete Handout 2.2: Graphing Quadratic Functions Where b≠ 0 independently.

Upon class completion, allow two students to display their work for any two functions from section 1. Then, briefly discuss sample responses for sections #2 - 4 as a class. Verify all graphical features using the calculator. Instruct students to place this document in their binder.

Note: Given time, use the formula for calculating the x- coordinate of the vertex $x = \frac{-b}{2a}$ using examples from the Handout.

Reflection and Closing: Bringing it All Together

Distribute one copy of **Handout 2.3: The Conjecture's All Mine** and one sheet of lined a Post-It Note to each student. Read the directions aloud. Explain that some of the graphs are from the Anticipatory Set activity. Upon class completion, instruct students to turn the document in. Review their answers.

Note: If time permits, take a few moments to ask students to review their responses on the bottom of the chart paper from the Anticipatory Set. Were any of their guesses correct (SMP.1-8)?

Page | 57

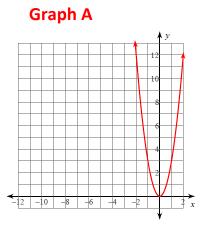
Homework Give each student one domino and two sheets of lined Post-It Notes. Explain that one side of the domino indicates the total number of things they have learned over the past few days' lessons. The other side of the domino indicates the total number of people in their family they must share that information with. They will record their work on each Post-it Note (SMP.1-8). Tell 2 family members Tell 5 family members 5 things that you learned over 2 things that you learned over or the past few days' lessons. the past few days' lessons. For students who are EL, have disabilities, or perform well below grade level: Email their parents the instructions for this homework a week in advance. Attach the URL for • Geogebra, Exploring Quadratic Functions, and encourage them to work with their child to use the "slider" application to see how the graph of the parent function, $y = x^2$, changes when the values of "a", "b", and "c" change independently or simultaneously. Extensions for students with high interest or working above grade level: Give students a Domino with one side of the Domino blank. Tell them that the blank side of the Domino indicates that they can illustrate /draw what they learned over the past few days' lessons, and the numbered side of the Domino indicates the total number of people in their family they must share their illustration with.

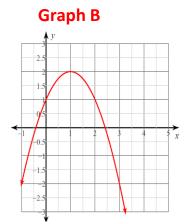
Handout 2.1: Say It with a Venn

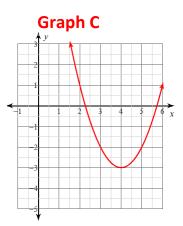
Directions:

As a group:

- 1. examine the three graphs, $f(x) = ax^2 + bx + c$ shown below.
- 2. using what you learned in the previous day's lesson, compare and contrast the 3 graphs using a Venn Diagram.
- 3. write **one guess** about the value (i.e. positive or negative) of *a*, *b*, and *c* for each function at the bottom of your chart paper.







Handout 2.2: Graphing Quadratic Functions Where b ≠ 0

Name

Date _____

- Use a graphing calculator to sketch the graph for each function in the table below. Adjust the viewing window accordingly to ensure the graph fits on the screen. For the last four functions, start with a sketch of the parent function in pencil. Be sure to write each function in standard form first, if needed.
 - a. Use a colored pencil (or highlighter) to emphasize the vertex and axis of symmetry.
 - b. Use a different color pencil (or highlighter) to emphasize the *x*-intercept.
 - c. Use a different color pencil (or highlighter) to emphasize the *y*-intercept.

Function	Sketch	Value of <i>b</i> in the function	Coordinates of the vertex	Equation for the axis of symmetry
<i>y=x</i> ²				
y=x ² + 12				
y - 12=x ² - 4x				
y – x ² – 11x=12				
y + 8.8x = x ² + 12				

Handout 2.2: Graphing Quadratic Functions Where b ≠ 0

- Is the effect of changing the value of "b" the same if "a" < 0? Provide a detailed explanation by using your calculator to analyze the graphs of the following functions to find out.

$$y = -1x^{2} - 4x$$

$$y = -1x^{2} + 2x$$

$$f(x) = -1x^{2} + 6x$$

$$h(t) = -t^{2} - 4t$$

* $h(t) = -t^{2} - 4t + 2$

Some questions adapted from "Graphing Calculator Activities" (Revised Edition) Dale Seymour Publications

Handout 2.2: Graphing Quadratic Functions Where b ≠ KEY

Name

Date____

- Use a graphing calculator to sketch the graph for each function in the table below. Adjust the viewing window accordingly to ensure the graph fits on the screen. For the last four functions, start with a sketch of the parent function in pencil. Be sure to write each function in standard form first, if needed.
 - a. a colored pencil (or highlighter) to emphasize the vertex and axis of symmetry.
 - b. Use a different color pencil (or highlighter) to emphasize the *x*-intercept.
 - c. Use a different color pencil (or highlighter) to emphasize the *y*-intercept.

Function	Sketch	Value of <i>b</i> in the function	Coordinates of the vertex	Equation for the axis of symmetry
y=x²		0	(0,0)	x = 0
y=x²+12		12	(0, 12)	x = 0
y-12=x ² 4x		12	(2,8)	x = 2
y-x ² -11x=12	-70	12	(-5.5,-18.25)	x = -5.5
y+8.8x=x ² +12		12	(4.4, -7.34)	x = 4.4

Handout 2.2: Graphing Quadratic Functions Where b ≠ 0

2. How does the value of "b" affect the graphs of parabolas of the form $y = ax^2 + bx + c$?

It shifts the parabola left and right of the parent function.

3. What is the difference between the graphs of parabolas with positive values of "b" and the graphs of parabolas with negative values of "b?"

Negative "b" values shift it to the right and positive "b" values shift it to the left.

4. Is the effect of changing the value of "b" the same if "a" < 0? Provide a detailed explanation by using your calculator to analyze the graphs of the following functions to find out.

$$y = -1x^{2} - 4x$$

$$y = -1x^{2} + 2x$$

$$f(x) = -1x^{2} + 6x$$

$$h(t) = -t^{2} - 4t$$

* $h(t) = -t^{2} - 4t + 2$

A negative "a" flips the parabola upside down and the negative "b" values still make it shift left and right but with a negative "a" the shifts go the opposite way from before (negatives now go to the left and positives to the right)

Some questions adapted from "Graphing Calculator Activities" (Revised Edition) Dale Seymour Publications

Handout 2.3: The Conjecture's All Mine

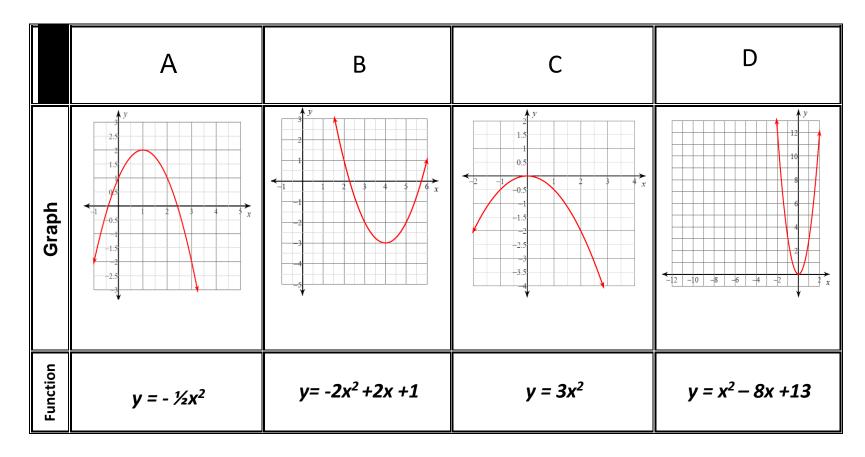
Name_____

Date _____

Directions:

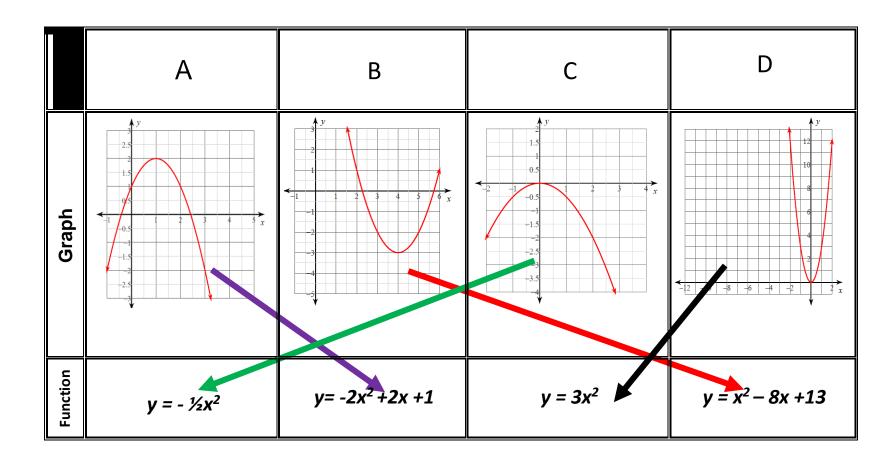
You may NOT use a calculator on this handout.

- 1. Using what you learned over the past two class days, draw a line to match each graph with its function rule.
- 2. Provide a short rationale for your selections on a lined Post-it note. Stick it to the top of this handout upon completion.
- 3. Be sure to use academic vocabulary in your response.



Handout 2.3: The Conjecture's All Mine





Lesson 3: Linear Factors and Standard Form

Focus Standard: A-APR.3

Additional Standards: F-IF.1, F-IF.4, F-IF.7a, F-IF.9

Standards for Mathematical Practice: SMP.2, SMP.3, SMP.4, SMP. 6, SMP.7, SMP.8

Estimated Time: 60 minutes – 180 minutes

Resources and Materials:

- Bell or grade-appropriate music
- Card stock
- Chart paper
- Colored pencils or highlighters
- Index card
- Markers
- Sticky note (optional)
- Questions 4 Quadratics (Q4Q) and Answers 4 Quadratics (A4Q) Wall
- TI-83/TI-84
- Handout 3.1: Entrance Ticket
- Handout 3.2: Four Coordinate Planes (Note: fold in half, print front to back)
- Handout 3.3: From Factoring to Standard and Back Again
- Handout 3.4: Spin the Frayer
- Desmos Graphing Calculator: www.Desmos.com

Lesson Targets:

- Students will solve quadratic equations by factoring.
- Students will graph quadratic functions and know that their roots are synonymous with their x-intercepts/zeros.

Guiding Questions:

- What is the relationship between the x-intercepts, factors, roots, and zeros of a quadratic function?
- What information is easily found given the factored form of a quadratic function?
- Are all quadratic functions factorable? Justify your response.

V	/ocabulary
Academic Vocabulary: Coordinate Point(s) Factors of Zero Property/Zero Product Property Linear Factor Maximum Minimum Parabola Parent Function Quadratic Function Range Roots Satisfies Solution Standard Form Vertex x-intercept y-intercept Zeros	 Instructional Strategies for Academic Vocabulary: Introduce academic vocabulary with student-friendly definitions and pictures Model how to use academic vocabulary in discussion Discuss the meaning of an academic vocabulary word in a mathematical context Justify responses and critique the reasoning of others algebraically, geometrically, and/or technologically using academic vocabulary Create pictures/symbols to represent academic vocabulary Write or use literacy strategies involving academic vocabulary

Symbol	Type of Text and Interpretation of Symbol
	Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level
~	Assessment (Pre-assessment, Formative, Self, or Summative)
	Instructional Plan
	nts to explore factorizations that are available as stated in the standard. Teachers should consider quent lessons on the standards A-REI.4 and F-IF.8
	The and Student Outcomes: Students will use the Zero Product Property to identify the zeros of $rm y = (ax + b)(cx + d)$ and $y = ax^2 + bx + c$, and use the zeros to construct a rough draft of the function.
anecdotal data throughout the	on, print Handout 3.1: Entrance Ticket on cardstock and cut out each ticket. You will need to collect e lesson today to determine which ticket you will give each student at the end of class. Collect data tudent questions, and work samples as they work independently or with other students.

Anticipatory Set: Exploring Factored Form of Functions

Distribute **Handout 3.2: Four Coordinate Planes** to each student and ask them to sketch separate graphs of the functions y = (x - 4)(x - 2) and y = (x - 3)(x - 5) on one half of the front of their handout. Encourage students to:

- use the table function on their calculator to create a table of values.
- record the table of values in the margin closest to the graph.
- label their graphs as **A** and **B** and label the axes appropriately.
- identify all key features of the graph and the table using a colored pencil or highlighter.

Note: Consider replacing the variable "y" with the number "0" in one of the functions to spark conversation about why it is appropriate to do this when graphing.

Ask students:

- ✓ What observations do you make about the function rule and the graph?
- \checkmark Is there evidence in the table that supports your observation?

Instruct students to **clear out** their calculators and to sketch separate graphs of the functions $y = x^2 - 6x + 8$ and $y = x^2 - 8x + 15$ on the **other half** of the front their handout. Encourage students to:

- use the table function on their calculator to create a table of values.
- record the table of values in the margin closest to the graph.
- label their graphs as **C** and **D** and label the axes appropriately.
- identify all key features of the graph and the table using a colored pencil or highlighter.

Note: Consider writing one of the functions in function notation and the other in standard form to continue to activate prior knowledge.

Graphsand	are	Graphs	and	ar	e
The tables for Graphs	and	_are The	e tables for Graphs	and	_are
Therefore, I can ass	ume that	and I can prove it by	(hint: "property")	"property").	Here is my proof:
 To help them fill i 	-	ies, or perform well below a rovide them with a post-it n	-	the following	words on
it and explain tha independently. B	in the blanks, p It this is not a c e sure to mix th	rovide them with a post-it n omplete list and there are so ne order of the words listed nay not be used more than o	ote or index card with ome blanks they will h below prior to giving t	nave to come	up with
it and explain tha independently. B	in the blanks, p It this is not a c e sure to mix th	rovide them with a post-it n omplete list and there are so ne order of the words listed	ote or index card with ome blanks they will h below prior to giving t nce.	nave to come	up with udent and
it and explain tha independently. B inform them that	in the blanks, p It this is not a c e sure to mix th	rovide them with a post-it n omplete list and there are so ne order of the words listed nay not be used more than c	ote or index card with ome blanks they will h below prior to giving t once.	have to come to the st	up with udent and etry
it and explain tha independently. B inform them that	in the blanks, p It this is not a c e sure to mix th	rovide them with a post-it n omplete list and there are so ne order of the words listed nay not be used more than o y-intercept	ote or index card with ome blanks they will h below prior to giving t once.	have to come of them to the st axis of symmetry	up with udent and etry
it and explain tha independently. B inform them that	in the blanks, p It this is not a c e sure to mix th	rovide them with a post-it n omplete list and there are so ne order of the words listed nay not be used more than o y-intercept vertex	ote or index card with ome blanks they will h below prior to giving t once.	nave to come of them to the st axis of symme equivalent/th	up with udent and etry ne same

Have students Turn and Talk to a classmate about their response to the writing prompt above. Upon completion share a possible correct response and ask a student to translate their observations into a "Question" for the Q4Q Wall and instruct another student to translate their observations into an "Answer" for the A4Q Wall (SMP.3).

Note: Be sure student responses include the academic vocabulary "distributive property" (the acronym "F.O.I.L." may assist some students) and "standard form" (SMP.6).

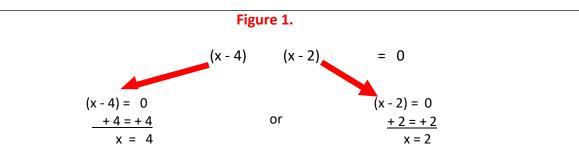
Activity 1: Understanding the Zero Product Property

Explain to students that what they have just explored is the factored form of function. Remind them what a "factor" is as it relates to what they learned in elementary school about "factors" and multiplication. Progress by saying that the difference is each "factor" is not a single number, rather they are called "linear factors." Ask the following question: "*Why do you think they are called 'linear factors'?*" Guide students into understanding that each separate factor has a polynomial with a degree of 1 (SMP.2, SMP.7, SMP.8).

Re-display the function rule y = (x - 4) (x - 2) on the Smart Board and instruct students to replace the variable "y" with "0"—creating an equation. Tell them to Turn and Talk to their neighbor about each of the questions below. Be sure to encourage them to use their notebook paper to brainstorm and to ask clarifying questions if the they do not agree with the response provided by their classmate (SMP.3). Upon completion ask a few students to share out the answers to the following questions:

- ✓ Can you explain in layman's terms and algebraically what this equation means?
- \checkmark Can you be sure what the value of "x" equals in the first linear factor?
- ✓ Can you be sure what the value of "x" equals in the second linear factor?
- ✓ Is at least one of the factors equal to zero?
- ✓ Is there a way to guarantee there is a solution?

Explain that at least one of the factors must be equal to zero. Model for the students how to find the solutions of the equation (x - 4)(x - 2) = 0 using the Factors of Zero Property (or Zero Product Property) and using the equal sign consistently and appropriately throughout (SMP. 6) [Figure 1].



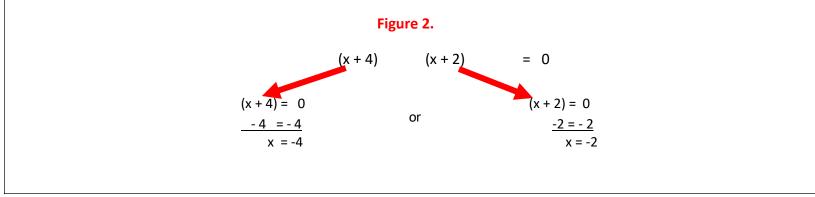
Use the property of substitution to validate the solutions.

Repeat this process for the equation/function 0 = (x - 3) (x - 5) as well. Discuss with students the connection that exists between both linear factors and the zeros (x-intercepts) of the graph. Instruct students to write a note in their own words about what they just observed. Allow a few students to share.

Note: Pay close attention to students' responses as they work with the negative constants in each parenthesis.

Instruct students to turn their Handout over and to sketch separate graphs of the functions y = (x + 4) (x + 2) and y = (x + 3) (x + 5) on one half of the back of their Handout.

Model for the students how to find the solutions of the equation 0 = (x + 4) (x + 2) using the Factors of Zero Property (or Zero Product Property) and the use of using the equal sign consistently and appropriately throughout (SMP. 6) [Figure 2].



Use the property of substitution to validate the solutions.

Repeat this process for the equation/function y = (x + 3) (x + 5) as well.

Instruct students to graph $y=x^2+6x+8$ $y=x^2+8x$ +15. Then, instruct them to write a note in their own words about what they just observed in these four examples.

Ask students to make a conjecture about the linear factors and the x-intercepts given the factored for $(x \pm r) (x \pm s) = 0$. Allow a few students come to the front of the room to justify their response using any values for "r" and "s." Use the property of substitution to validate their solutions.

Upon completion, ask a student to translate the classes' observations into a "question" for the Q4Q Wall and instruct another student to translate their observations into an "Answer" for the A4Q Wall.

Note: Consider placing an imaginary 1 in the front of the linear factors to facilitate the next few examples.

Take a few minutes to prove visually and solidify the fact that the factored form of a function and the standard form of each function are equivalent by graphing them on the same Coordinate Plane. Encourage students to change the graphing style to make one graph darker (as taught in the previous days' lesson).

Ask the following prompting question:

✓ Do you think the process for finding the x-intercepts and linear functions is the same when the leading coefficient ≠ 1?

Instruct students to use the last two Coordinate Planes on their handout and to sketch separate graphs of the functions y = (2x + 4) (x - 1) and y = (3x + 1) (x + 5). Model for students how to find the solutions for both functions using the Zero Product Property as before.

Note: Work closely with all students as they work with the fraction in both equations. Provide additional examples where "r" and "s" are Real Numbers and the coefficient of each "x" variable \neq 1. Ensure students graph all functions to verify their work. Use the property of substitution to validate all solutions.

Activity 2: Practice on Factoring

Distribute **Handout 3.3: From Factoring to Standard and Back Again.** Instruct students to work independently. Upon completion, share the correct answers and allow students to make corrections as necessary.

Note: This handout has several prompting questions to guide student thinking and self-discovery of transferring what they have just learned to assist them in writing the standard form of a quadratic function in factored form. Be sure to work with individual students throughout this activity (SMP.1, SMP.4).

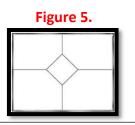
Reflection and Closing: Spin the Frayer

Have students form a group of four with the students closest to them. Provide each group with a sheet of chart paper and a few markers. Have students identify the student with the most vowels in their first and last name (combined) to draw the Modified Frayer Model below [Figure 5]. Display Handout 3.4: Spin the Frayer on the overhead.

Note: You may consider making a copy of the table on this document and require each group to glue it in the center of their Frayer Model.

Instruct students to write down one of the functions to start on in their "corner." For each function listed, students should fill in the circle that indicates whether the number shown represents the value of one of the x-intercepts/zeros of the function. Sound a bell or play music signaling them to rotate the Frayer Model counter-clockwise and critique each other's work, and in some instances, complete it. The goal is to be the first team to respond to the test question correctly by the time runs out.

Select one person from each group to be the spokesperson to share out to the entire class.



Homework

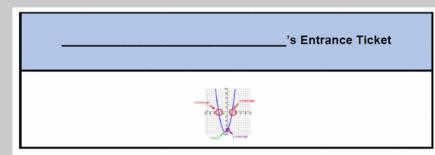
Based on any observations and anecdotal data you collected as you worked with students during today's lesson and listened to their responses, provide each student with one ticket (see "Entrance Ticket") as they walk out the door. Collect them at the door upon student arrival the next class day.

Indicate that they have three options to choose from as they respond to the item on the front of their ticket. Display the three options at the front of the room and provide an example of each one, if necessary.

- ✓ Something they learned from the topics discussed in this unit.
- \checkmark A question they have about the topics discussed in this unit.
- \checkmark A question they think someone else might have about the topics discussed in this unit.

For students who are EL, have disabilities, or perform well below grade level:

• Provide students that still appear to struggle the Entrance Ticket shown here. Encourage them to write as much as they know about each feature and relate it to what they learned today about linear factors, zeros, and roots.



Extensions for students with high interest or working above grade level:

• Give students two cards, one of which includes the Entrance Ticket shown here. Encourage students to use the calculator to identify the key features of the graph and make as many conjectures as they can. Explain that in the next few days you will retrieve the card from them and they will talk about this function form as a class.

Note: Vertex form is not included in this unit, but should be addressed in a subsequent unit.

_____'s Entrance Ticket
$$y = \frac{1}{2}(x-4)^2 - 2$$

's Entrance Ticket

Standard Form of a Quadratic Function

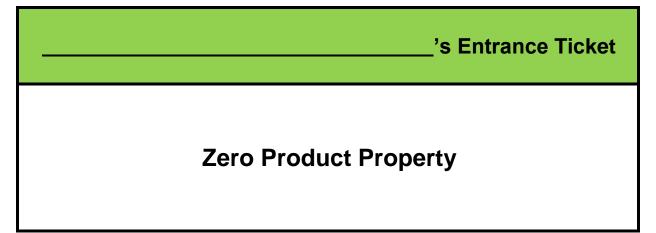
's Entrance Ticket

Using the calculator to evaluate the graph of a quadratic function

's Entrance Ticket

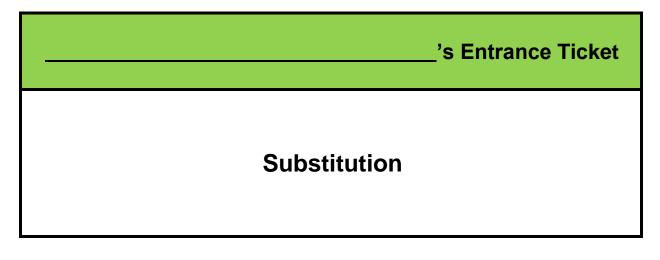
Linear Factors

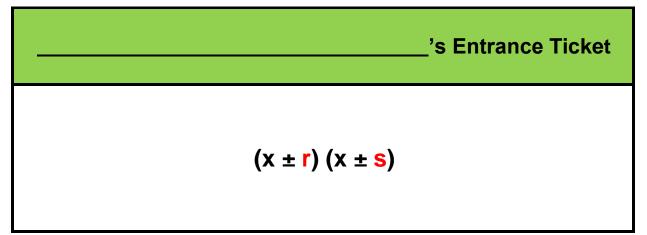
<u>'s Entrance Ticket</u> Given a quadratic function in factored form, how do you determine the location for the xintercept(s)?

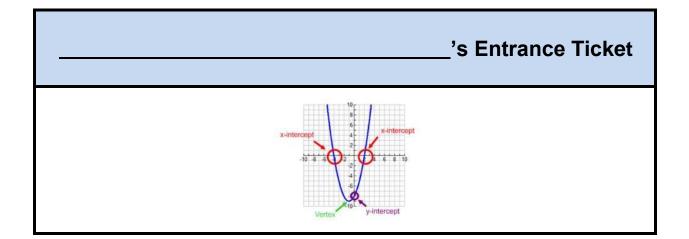


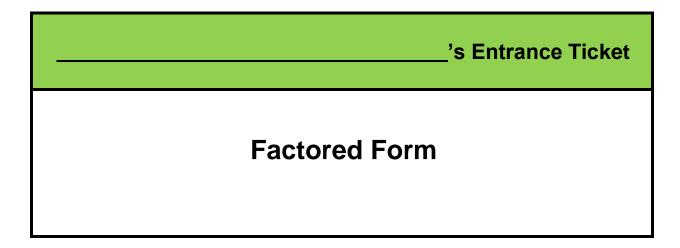
's Entrance Ticket

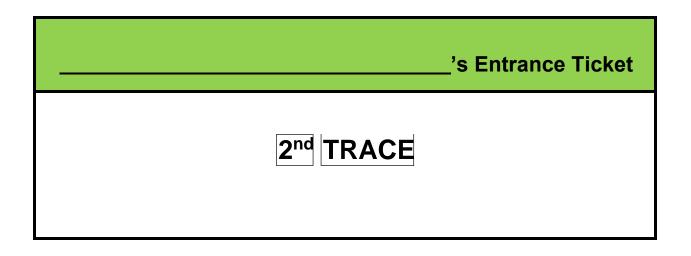
Roots, Zeros, and x-intercepts





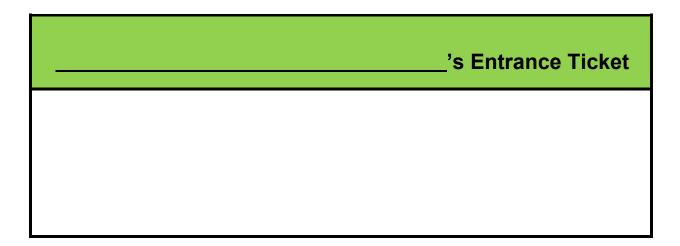






_____'s Entrance Ticket
$$y = \frac{1}{2}(x-4)^2 - 2$$

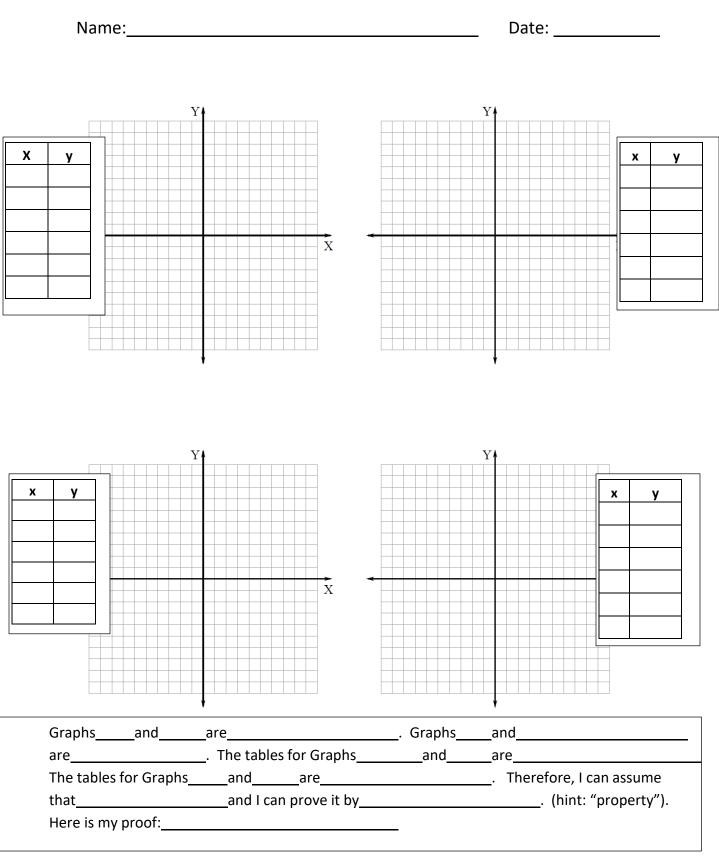
Create additional tickets based on your students' needs.

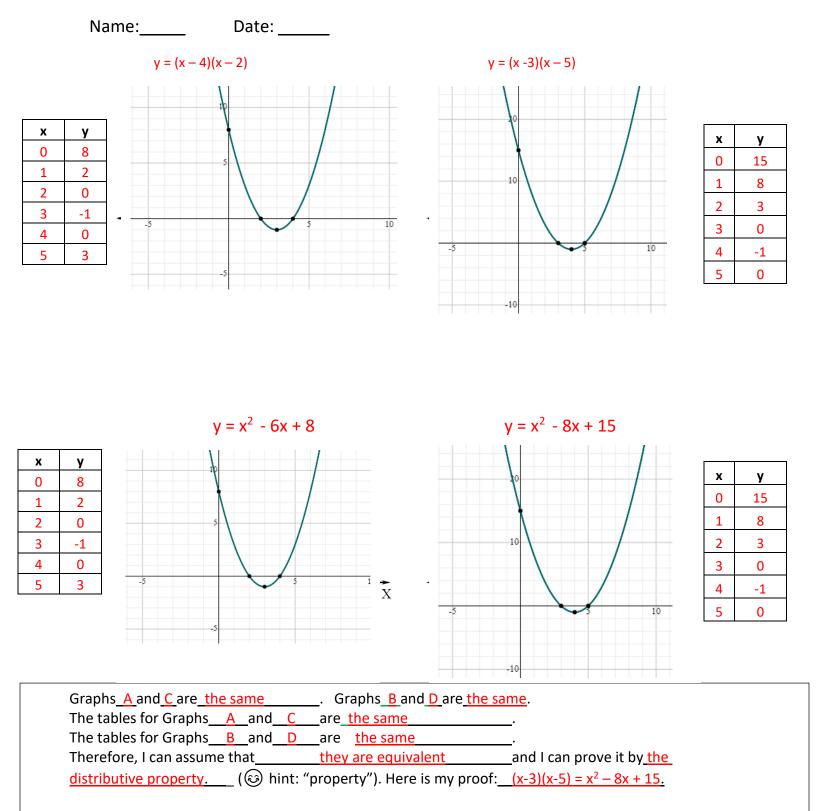


's Entrance Ticket

_____'s Entrance Ticket







Handout 3.2: Four Coordinate Planes -KEY

Handout 3.3: From Factoring to Standard and Back Again – Key

Name_____

Date _____

PART A

Directions: Complete the chart below using what you learned in class today. The first one has been done for you as an example. Attach all scratch paper.

	Linear Factors (x ± r) (x ± s)	What are the values of r and s?	Use the Distributive Property to Expand the Function to Standard Form	Calculate r + s (r) (s)	Solutions to the Function
А	(x – 3) (x + 2) = 0	r= -3 s = 2	$x^{2} + 2x - 3x - 6 = 0$ simplifies to $x^{2} - x - 6 = 0$	-3 + 2 = -1 (-3)(2) = -6	x = 3 or x=-2
В	(x-1)(x+4)=0	r = 4 s = -1	$x^{2} + 4x - x - 4 = 0$ simplifies to $x^{2} + 3x - 4 = 0$	4 + -1 = 3 (4)(-1) = -4	x = -4 or x = 1
C	(x + 2) (x + 4) = 0	r = 2 s = 4	$x^{2} + 4x + 2x + 8 = 0$ simplifies to $x^{2} + 6x + 8 = 0$	4 + 2 = 6 (4)(2) = 8	x = -4 or x=-2
D	(x-3)(x-2)=0	r = -3 s = -2	$x^2 - 2x - 3x + 6 = 0$ simplifies to $x^2 - 5x + 6 = 0$	-3 + - 2 = - 5 (-3)(-2) = 6	x = 3 or x=2
Ε	(x-1)(x-6) = 0	r = -1 s = -6	$x^2 - 6x - x + 6 = 0$ simplifies to $x^2 - 7x + 6 = 0$	-6 + -1 = -7 (-6)(-1) = 6	x = 1 or x= 6
F	(2x-1)(x-3)=0	r = - ½ s = - 3	$2x^2 - 6x - x + 3 = 0$ simplifies to $2x^2 - 7x + 3 = 0$	-6 + -1 = -7 (3)(1) = 3	x = 3 or x= ½
G	(4x – 3) (2x + 5) = 0	r = - ¾ s = 5/2	8x ² + 20x - 6x - 15 = 0 simplifies to 8x ² +14x - 15 = 0	20 + - 6 = 14 (-3)(5) = -15	x = - 3/4 or x= - 5/2

Handout 3.3: From Factoring to Standard and Back Again - Key

PART B

Directions: Use your responses from **Part A** to complete the following table. Attach all scratch paper.

	Standard Form	What are the values of r and s?	Calculate	Linear Factors (x ± r) (x ± s)
A	$\mathbf{x}^2 - \mathbf{x} - 6 = 0$	r= -3 s = 2	-3 + 2 = -1 (-3)(2) = -6	$x^{2}-3x+2x-6=0$ x(x-3) + 2 (x - 3) = 0 (x + 2) (x - 3)
В	$x^2 + 3x - 4 = 0$	r= 4 s = -1	4 +- 1 = -1 (-3)(2) = -6	$x^{2}-3x+2x-6=0$ x(x-3)+2(x-3)=0 (x+2)(x-3)
C	$x^2 + 6x + 8 = 0$	r = 2 s = 4	-3 + 2 = -1 (-3)(2) = -6	$x^{2}-3x + 2x - 6 = 0$ x(x - 3) + 2 (x - 3) = 0 (x + 2) (x - 3)
D	$x^2 - 5x + 6 = 0$	r= - 3 s = - 2	-3 + 2 = -1 (-3)(2) = -6	$x^{2}-3x + 2x - 6 = 0$ x(x - 3) + 2 (x - 3) = 0 (x + 2) (x - 3)
E	$x^2 - 7x + 6 = 0$	r= - 1 s = - 6	-3 + 2 = -1 (-3)(2) = -6	$x^{2}-3x + 2x - 6 = 0$ x(x - 3) + 2 (x - 3) = 0 (x + 2) (x - 3)
F	$2x^2 - 7x + 3 = 0$	r= - ½ s = - 3	-3 + 2 = -1 (-3)(2) = -6	$x^{2}-3x + 2x - 6 = 0$ x(x - 3) + 2 (x - 3) = 0 (x + 2) (x - 3)
G	$8x^2 - 14x - 15 = 0$	r= - ¾ s = 5/2	-3 + 2 = -1 (-3)(2) = -6	$x^{2}-3x+2x-6=0$ x(x-3) + 2 (x - 3) = 0 (x + 2) (x - 3)

Handout 3.3: From Factoring to Standard and Back Again

PART C: Factor and solve each function below using **r** and **s**. Check each solution using your calculator.

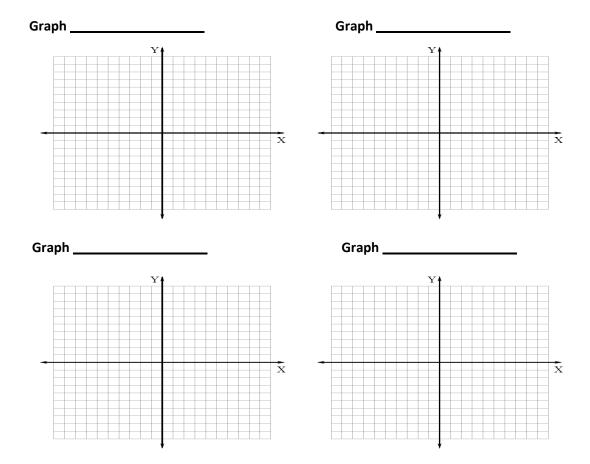
•	$6x^2 - 13x - 15 = 0$	(6x + 5)(x - 3) = 0	r = -5/6, s = 3
•	$4x^2 - 14x + 6 = 0$	(4x-2)(x-3)=0	r = ½ , s = 3
•	$4x^2 - 9 = 0$	(2x-3)(2x+3)=0	r = 3/2, s = -3/2
•	$4x^2 + 12x + 9 = 0$	(2x + 3)(2x + 3) = 0	r = -3/2
•	$12x^2 + 11x + 2 = 0$	(4x + 1)(3x + 2) = 0	r = -1/4 , s = -2/3
•	18x ² –37x +15 =0	(9x-5)(2x-3)=0	r = 5/9, s = 3/2
•	$9x^2 + 14x - 8 = 0$	(9x-4)(x+2)=0	r = 4/9, s = -2
•	$11x^2 - 35x + 6 = 0$	(11x-2)(x-3)=0	r = 2/11, s = 3

Handout 3.3: From Factoring to Standard and Back Again

PART D:

Select two functions from PART B and two functions from Part C and graph them on the coordinate planes below. Be sure to show all key features of the graph.

Verify the x-coordinate for the vertex using the equation $x = \frac{-b}{2a}$. Attach all scratch paper.



Handout 3.4: Spin the Frayer

Directions: For each function listed below, fill in the circle that indicates whether the number shown represents the value of one of the x-intercepts/zeros of the function.

	-3	-2	-1	1	2
$y = x^2 + 5x + 6$	0	0	0	0	0
$y = x^2 + x - 6$	0	0	0	0	0
$y = x^2 + 6x + 9$	0	0	0	0	0
Challenge Problem: y = 2x ² - 2x - 4	0	0	0	0	0

Adapted from the 2016-2017 MAAP/Questar Sample Test. Item #31

Directions: For each function listed below, fill in the circle that indicates whether the number shown represents the value of one of the x-intercepts/zeros of the function.

	-3	-2	-1	1	2
$y = x^2 + 5x + 6$			0	0	0
$y = x^2 + x - 6$		0	0	0	
$y = x^2 + 6x + 9$		0	0	0	0
Challenge Problem: y = 2x ² - 2x - 4	0	0		0	

Adapted from the 2016-2017 MAAP/Questar Sample Test. Item #31

Lesson 4: The Graph Tells It All

Focus Standards: A-APR.3, F-IF.4, F-IF.7a

Additional Standards: F-IF.5, F-IF.9

Standards for Mathematical Practice: SMP.1, SMP.2, SMP.3, SMP.4, SMP. 6

Estimated Time: 60 minutes – 180 minutes

Resources and Materials:

- Entrance Tickets (given out during the previous day's lesson)
- Card stock
- Colored pencils or highlighters
- Scissors
- Questions 4 Quadratics (Q4Q) and Answers 4 Quadratics (A4Q) Wall
- Handout 4.1: The Graph Tells It All
- Handout 4.2: Choose a Side

Lesson Target:

• Students will write the equation for a quadratic function given only the graph.

Guiding Questions:

- What is the most important information we need to write the function rule for a quadratic function?
- Are there any graphical features that are not used when attempting to write the function rule that models it?

Vocabulary			
Academic Vocabulary: • Factors of Zero Property/Zero Product Property • Linear Factor • Maximum/Minimum • Parabola • Quadratic Function • Roots/Solution/X-intercept/Zero • Standard Form • Vertex • y-intercept		 Instructional Strategies for Academic Vocabulary: Introduce academic vocabulary with student-friendly definitions and pictures Model how to use academic vocabulary in discussion Discuss the meaning of an academic vocabulary word in a mathematical context Justify responses and critique the reasoning of others algebraically, geometrically, and/or technologically using academic vocabulary Create pictures/symbols to represent academic vocabulary Write or use literacy strategies involving academic vocabulary 	
Symbol	Type of	Text and Interpretation of Symbol	
	Instructional support and/or extension suggestions for students who are EL, have disabilities, perform well below the grade level and/or for students who perform well above grade level		
\checkmark	Assessment (Pre-assessment, Fo	ormative, Self, or Summative)	
	Instructi	ional Plan	
Note: This lesson allows students to explore factorizations that are available as stated in the standard. Teachers should consider following this unit with subsequent lessons on the standards A-REI.4 and F-IF.8.			
Understanding Lesson Purpose and Student Outcomes: Students will combine all skills they learned over the past few days to learn how to write the function rule (or symbolic form) for a quadratic function that has been graphed in the Coordinate Plane.			
Note: Collect each student's "Entrance Ticket" they were given at the end of each class for homework. Take a few minutes to read each of their responses on the back and provide feedback to the entire class without providing each student's name. Upon completion, place tape on each Ticket and placed it on one of the whiteboards in random order.			

Anticipatory Set/Introduction to the Lesson: Story Time

Instruct the students that they are going to make up a quick math story, poem, or rap using the concepts that are printed on the front of each ticket. The story, poem, or rap can be fictional or real. The goal is to use as many tickets as possible to create one cohesive short story, poem, or rap. Encourage students to use their academic vocabulary throughout (SMP.6).

✓ To begin, allow one student to take one word off the whiteboard and provide the introduction. S/he writes their introduction on the other whiteboard, taping the Entrance Ticket where it belongs in the story. Another student should come to the whiteboard to retrieve a ticket and add to the story/poem/rap, and write their part immediately behind the previous student's work. They will also tape their entrance ticket where it belongs in the story/poem/rap. This process should continue until each student has had an opportunity to add to the story/poem/rap. The result will be an interactive word wall. Read the final product as a class and allow students to copy the it on notebook paper and discuss as a class.

For students who are EL, have disabilities, or perform well below grade level:

• Allow students to select their own original "Entrance Ticket" back and use it in this activity.

Activity 1: The Graph Tells It All

Display and distribute **Handout 4.1: The Graph Tells It All** to each student and explain that you will go over this handout as a class to ensure precision. Upon completion, ask a student to translate the classes' observations into a "Question" for the Q4Q Wall and instruct another student to translate their observations into an "Answer" for the A4Q Wall.

Note: Be sure student responses include the academic vocabulary (SMP.6).

For students who are EL, have disabilities, or perform well below grade level:

• Allow students to reference any notes they have from the previous day's class as needed.

Reflection and Closing: All Hands On Deck

Tell each student to take out a sheet of card stock and trace their hand. Instruct them to complete the following prompts on each finger as it relates to today's lesson.

- ✓ One thing I am confident about is ______
- ✓ When writing the function rule for a quadratic I have to remember ______
- ✓ Tonight, as I do my homework, I will "tri"_____
- ✓ Before solving any problem, I ______
- ✓ As I thumb through my notes, I noticed ______

Upon completion, allow students to stand in a circle as a whole class and read them aloud.

Note: You may consider posting these up in the classroom, hanging them from ceiling, or taping them around the perimeter of the windows/doors.

Homework

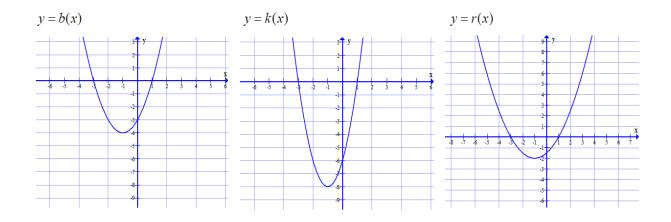
Distribute **Handout 4.2: Choose A Side** for homework and encourage students to complete this activity without a calculator (SMP.1-4 and SMP.6).

Handout 4.1: The Graph Tells It All

Name_____ Date _____

Directions: Complete each item provided without the use of a calculator.

1. Let's look at the following three graphs of quadratic functions. Identify the functions zeros, linear factors, and vertex in the table below.



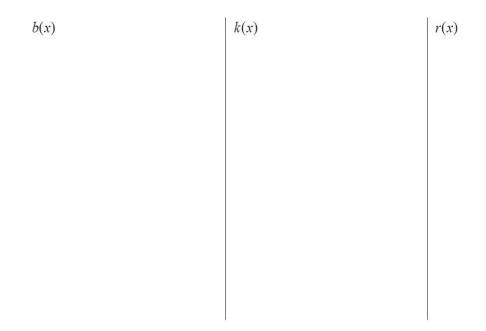
Function	Zeros	Linear Factors	Vertex
y = b(x)			
y = k(x)			
y = r(x)			

2. What is similar amongst all three graphs?

3. What is different amongst all three graphs?

Handout 4.1: The Graph Tells It All

4. The factored form of a quadratic function is y = a(x - p)(x - q). Using the vertex and the two linear factors that you discovered, find the value of *a* for b(x), k(x), and r(x).



5. Using the value of a, write the quadratic equation in factored form for the given graph of each quadratic function.

6. Explain the role that the value of "a" plays in the factored form of a quadratic function.

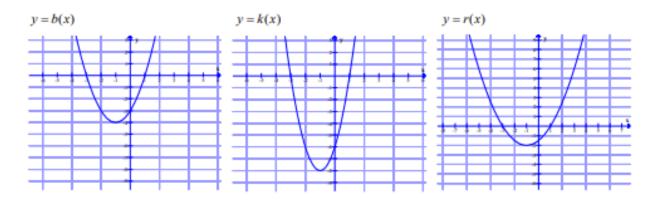
7. What do you think would happen to the graph of b(x) if the value of a is less than zero?

Adapted from CTUQC and Student Achievement Partners

Comparing Quadratics Function in Factored Form

Name KEY

1. Let's take a look at the following three graphs of quadratic functions. Identify the functions zeroes, linear factors, and vertex in the table below.



Function	Zeroes	Linear Factors	Vertex
y = b(x)	x = -3, 1	(x+3)(x-1)	(-1, -4)
y = k(x)	x = -3, 1	(x+3)(x-1)	(-1, -8)
y = r(x)	x = -3, 1	(x+3)(x-1)	(-1, -2)

2. What is similar amongst all three graphs?

The quadratic functions all have the same zeroes and linear factors.

3. What is different amongst all three graphs?

The quadratic functions all have different vertices.

4. The factored form of a quadratic function is y = a(x - p)(x - q). Using the vertex and the two linear factors that you discovered, find the value of *a* for b(x), k(x), and r(x).

$$b(x) \qquad k(x) \qquad k(x) \qquad r(x)$$

$$y = a(x - p)(x - q)$$

$$y = a(x + 3)(x - 1)$$

$$-4 = a(-1 + 3)(-1 - 1)$$

$$-4 = a(2)(-2)$$

$$-4 = -4a$$

$$1 = a$$

$$k(x) \qquad y = a(x - p)(x - q)$$

$$y = a(x + 3)(x - 1)$$

$$-8 = a(-1 + 3)(-1 - 1)$$

$$-8 = a(2)(-2)$$

$$-8 = -4a$$

$$2 = a$$

$$\frac{1}{2} = a$$

5. Using the value of a, write the quadratic equation in factored form for the given graph of each quadratic function.

$$b(x) = (x + 3)(x - 1)$$
 $k(x) = 2(x + 3)(x - 1)$ $r(x) = \frac{1}{2}(x + 3)(x - 1)$

6. Explain the role that the value of "a" plays in the factored form of a quadratic function.

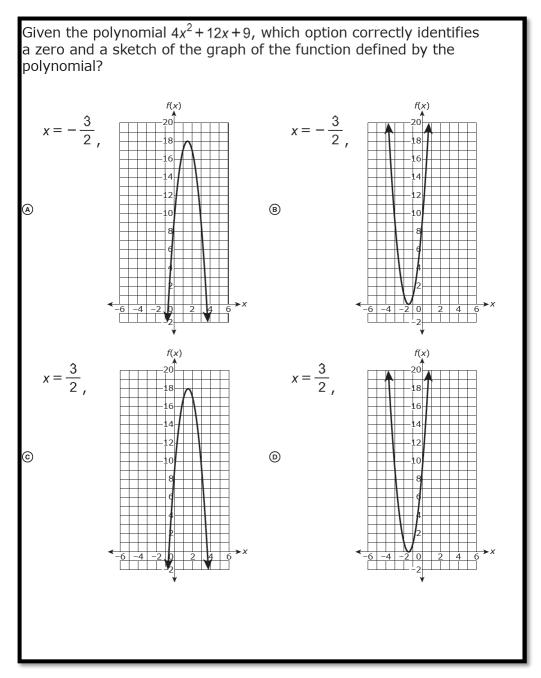
Possible Answer: The value of a stretches or shrinks the size of the graph.

7. What do you think would happen to the graph of b(x) if the value of a is negative?

Possible Answer: The parabola would reflect about the x-axis. The graph would flip upside down.

Handout 4.2: Choose a Side

Denise and Edward are having a disagreement. Denise states that she can answer the MAP Assessment problem below without a calculator. Edward states that it is impossible to do without a calculator. Whose side are you on? Justify your response.



Adapted from the 2016-2017 MAP/Questar Sample Test. Item #57

Lesson 5: Will He Make the Basket?

Focus Standard: F-IF.4

Additional Standard: F-IF.7a

Standards for Mathematical Practice: SMP.3, SMP.4, SMP.7

Estimated Time: 60 minutes

Resources and Materials:

- TI-83/TI-84
- Handout 4.1: Admit Slip
- Handout 4.2: Real World of Parabolas

Lesson Targets:

- Students will solve determine key features of quadratic equations.
- Students will interpret key features of quadratic equations in real-world context.

Guiding Questions:

- What is the relationship between the coefficient of a quadratic term of a quadratic equation and the graph of its function?
- What is the relationship between the key factors of the parabola and the quadratic equation it represents and the realworld situation it represents?

Vocal	oulary
Academic Vocabulary: Coordinate Point(s) Factors of Zero Property/Zero Product Property Linear Factor Maximum Minimum Parabola Quadratic Function Range Roots Satisfies Solution Standard Form Vertex x-intercept y-intercept Zeros	 Instructional Strategies for Academic Vocabulary: Introduce academic vocabulary with student-friendly definitions and pictures Model how to use academic vocabulary in discussion Discuss the meaning of an academic vocabulary word in a mathematical context Justify responses and critique the reasoning of others algebraically, geometrically, and/or technologically using academic vocabulary Create pictures/symbols to represent academic vocabulary Write or use literacy strategies involving academic vocabulary

Symbol	Type of Text and Interpretation of Symbol
	Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level
\checkmark	Assessment (Pre-assessment, Formative, Self, or Summative)
	Instructional Plan
 5.1: Admit Slip. 2. Before Class? Go to: htt Under the Takes, download to Show students the halftakes Discuss the students answers Show the students the fulltake If more time allows, show hat 3. Ask students to look at the students to think about what 4. Ask students to solve the of and by using the quadratic for 5. Ask students to plot the x- about the volleyball's path th 6. Ask students to find the y- 7. Ask students to plot the volleyball's plot the volleyball's 	tudents to see how well they can predict whether a basketball player can hit a basket. Give Handout <u>p://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/</u> he Alltakes zipfile. 3, 4, and 6 and ask students to make their predictions. 5 to the discussion questions. 5 to the discuss of other throws. 5 equation on the back of the admit slip. The equation shows the path of a volleyball hit over a net. Ask 5 the x- and y-axes represent in this situation. 1 quadratic equation to find the zeros of the function. Review solving by factoring, completing the square, 1 yrmula. 1 intercepts. Ask students to do a think-pair-share with a partner to discuss what the x-intercepts mean 1 yrough the air. 1 intercept and plot it. Discuss what that means about the path of the volleyball. 1 ertex. Review methods. 1 ertex. Ask students to do a think-pair-share with a partner to discuss what each value of the vertex

Handout 5.1 Admit Slip

Name:___

1. Watch each video and make a prediction about whether he will or will not make the shot.

	Will He Make the Shot?
Take 1	
Take 3	
Take 6	

- 2. What do you think?
 - A. Why does the ball need to curve to go in?
 - B. What do you think will be true about the coefficient of the quadratic term that would be part of the equation created by the graph of the shot?
- 3. Watch each video and decide if each prediction was correct.

	Was My Prediction Correct?
Take 1	
Take 3	
Take 6	

Handout 5.2: The Real World of Parabolas

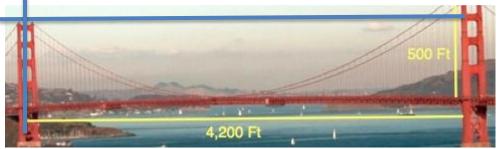
- 1. Look at the parabolic shape graphed in the picture of the launch from the catapult. The x-axis is feet from the starting point. The y-axis is seconds from launch.
 - a. What does the y-intercept (0, 5) mean for the catapult?
 - b. The vertex is approximately at (4, 10). What does the 4 mean for the catapult?
 - c. The vertex is approximately at (4, 10). What does the 10 mean for the catapult?
 - d. The x-intercept is about (10.5, 0). What does this mean about the catapult?
 - e. Do you think the coefficient of x² is positive or negative? Explain your reasoning.

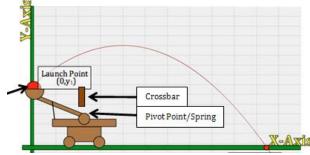
http://teachersinstitute.yale.edu/curriculum/units/2012/4/12.04.03.x.html

- 2. Look at the parabolic curve of the bridge shown below.
- a. The vertex is at approximately (2,100, -250). What does the -250 mean?
- b. The y-intercept is at the origin. Describe that in terms of the bridge.
- c. The x-intercept is (4,200, 0). What does the 4,200 mean about the bridge?

y-axis(top of brace is 0,0)

x-axis(feet from left brace)





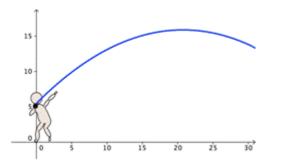
http://jwilson.coe.uga.edu/EMAT6680Su12/Gilles/Assignment2/gillesAssign2.html

- 3. The arch of this bridge is a parabolic curve. The origin is marked with a star. The x-axis is the number of feet from the left pillar and the y-axis is the number of feet above the water.
 - a. The vertex is at approximately (500, 350). What does the 500 tell about the arch?
 - b. The vertex is at approximately (500,350). What does the 350 tell about the arch?
 - c. The x-intercept is (1000,0). What does this tell you about the arch?



http://passyworldofmathematics.com/sydney-harbour-bridge-mathematics/

- 4. The parabola below shows the throw of a shot-putter. The y-axis is the feet above the ground and the y-axis is the feet from the shot-putter.
 - a. The y-intercept is (0,5). What does this mean about the throw?
 - b. The vertex is at approximately (20, 16). What does the 20 mean about the throw?
 - c. The vertex is at approximately (20, 16). What does the 16 mean about the throw?



MS Exemplar Unit
Mathematics

http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html

Handout 5.2: The Real World of Parabolas - Key

- 1. Look at the parabolic shape graphed in the picture of the launch of a ball from the catapult. The x-axis is feet from the starting point. The y-axis is feet above the ground.
- a. What does the y-intercept (0, 5) mean for the catapult? The catapult launched from 5 ft above ground.
- b. The vertex is approximately (4, 10). What does the 4 mean for the catapult?

The ball was 4 ft from the launch spot at its highest point.

c. The vertex is approximately at (4, 10). What does the 10

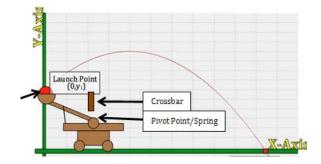
mean for the catapult? The ball was 10 ft above the ground at its highest point.

d. The x-intercept is about (10.5, 0). What does this mean

about the catapult? The ball hit the ground 10.5 feet from the launch spot.

http://teachersinstitute.yale.edu/curriculum/units/2012/4/12.04.03.x.html

2. Look at the parabolic curve of the bridge shown below.



- d. The vertex is at approximately (2,100, -250). What does the -250 mean? The lowest point is 250 ft below the top of the braces.
- e. The y-intercept is at the origin. Describe that in terms of the bridge. The top of the left brace is the beginning of the bridge wires.
- f. The x-intercept is (4,200, 0). What does the 4,200 mean about the bridge? The top of the right brace is 4,200 ft from the left brace.

y-axis(top of brace is 0,0)

x-axis(feet from left brace)

http://jwilson.coe.uga.edu/EMAT6680Su12/Gilles/Assignment2/gillesAssign2.html

- 3. The arch of this bridge is a parabolic curve. The origin is marked with a star. The x-axis is the number of feet from the left pillar and the y-axis is the number of feet above the water.
 - a. The vertex is at approximately (500, 350). What does the 500 tell about the arch?

The highest point of the arch is 500 feet over from the base of the left pillar.

b. The vertex is at approximately (500,350). What does the 350 tell about the arch?

The highest point is 350 feet above the water.

c. The x-intercept is (1000,0). What does this tell you about the arch? The bottom of the right arch base is 100 feet over from the base of the left pillar.



http://passyworldofmathematics.com/sydney-harbour-bridge-mathematics/

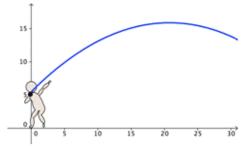
- 4. The parabola below shows the throw of a shot-putter. The y-axis is the feet above the ground and the y-axis is the feet from the shot-putter.
 - a. The y-intercept is at (0,5). What does this mean about the throw?

The shot-putt was thrown from a height of 5 feet.

b. The vertex is at approximately (20, 16). What does the 20 mean about the throw?

The highest point is 20 feet over from the shot-putter.

c. The vertex is at approximately (20, 16). What does the 16 mean about the throw? The highest point is 16 feet above the ground.



http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U10_L2_T1_text_container.html

Lesson 6 : Start Your Engines!		
Focus Standards: F-IF.4. F-IF.7a, F-IF.8a, and A-APR.3		
Standards for Mathematical Practice: SMP.1, SMP.2, SMP.3, SMP.4, SMP.5, SMP. 6, SMP.7, SMP.8		
Estimated Time: 60 minutes – 180 minutes		
Resources and Materials: 4-color spinner Colored Pencils/Crayons (at home) Construction paper (at home) Plastic or Paper Cups Glue (at home) Pennies Scissors (at home) Handout 6.1: Start your Engines! Handout 6.2: Performance Task Rubric Handout 6.3: Fun-Shapes Unit Summary		
 Lesson Target: Students will use their depth of knowledge of quadratic functions, their key features, and the relationship between those features and their real-world context. Guiding Question: What connections can be made between the linear factors of a quadratic function, the vertex, and the x-intercepts to the real-world situations they represent? 		

Vocabulary		
Academic Vocabulary: Note: Students are encouraged to use several of these words to construct their response for the Performance Task. The teacher should display these words on the Smart Board as students complete the Task. • Axis of Symmetry • Coordinate Point(s) • Decreasing • End Behavior • Equidistant • Function • Increasing • Linear Function • Maximum • Midpoint • Minimum • Parabola • Quadratic Function • Satisfies • Solution • Standard Form • Vertex • x-intercept • y-intercept	 Instructional Strategies for Academic Vocabulary: Introduce academic vocabulary with student-friendly definitions and pictures Model how to use academic vocabulary in discussion Discuss the meaning of an academic vocabulary word in a mathematical context Justify responses and critique the reasoning of others algebraically, geometrically, and/or technologically using academic vocabulary Create pictures/symbols to represent academic vocabulary Write or use literacy strategies involving academic vocabulary 	

Symbol	Type of Text and Interpretation of Symbol	
	Instructional support and/or extension suggestions for students who are EL, have disabilities, or perform well below the grade level and/or for students who perform well above grade level	
\checkmark	Assessment (Pre-assessment, Formative, Self, or Summative)	
Instructional Plan		
 Understanding Lesson Purpose and Student Outcomes: Students will utilize what they have learned about the key features of a quadratic function and the relationship of those features to real-world situations. Anticipatory Set/Introduction to the Lesson: Ask students to write down the answers to the following questions: What are two-real world examples of quadratic equations that would have a positive coefficient of the quadratic term? What are two-real world examples of quadratic equations that would have a negative coefficient of the quadratic term? If a graph can be represented by the equation y = - x² + 3x + 10, what is the y-intercept and what would it mean in real-world terms? 		

Activity 1: Performance Task

Distribute **Handout 6.1 : Start Your Engines** to each student. Instruct them to work independently to complete the task. Collect all work upon completion (SMP.1-8).

For students who are EL, have disabilities, or perform well below grade level:

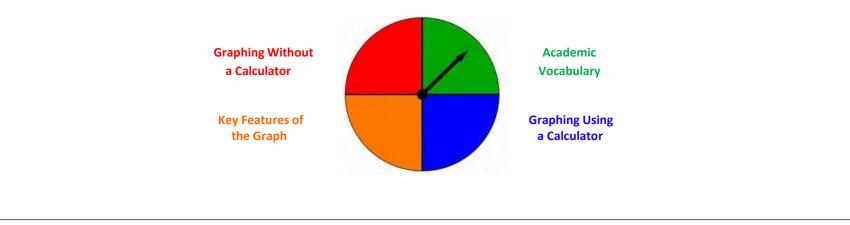
- Consider using some of the questions that have been used thought the unit (or a variation of them).
 - o How do you find the x-intercepts and what do they mean in real-life?
 - How do you find the y-intercepts and what does it mean in real-life?
 - \circ $\;$ How do you find the vertex and what does it mean in real-life?

Extensions for students with high interest or working above grade level:

• Ask students to write a new equation that would score a total of 0 points.

Reflection and Closing: Let the Spinner Decide

Instruct students to go to one of the four walls and stand in a circle. This will quickly form 4 large groups. Walk around to each group with a 4-color spinner and identify one student to spin. The color that the spinner lands on indicates the final conversation that each group will have related to the entire unit. Upon completion, have a few students share out (SMP.2-4 and SMP.6).



Homework

Extra Credit Project: Instruct students that they must select and cut out 5 shapes from the **Handout 5.2: Fun Shapes Unit Summary** that depict/illustrate their progression of learning over this unit. Encourage students to write in each shape, use cardstock/color paper, colored pencils/crayons, and other items to decorate their final product.

Note: (1) Shapes can be used multiple times/duplicated. (2) Determine your own point system for this project. (3) Display student work when completed. (4) Give students three school nights to complete this project.

Handout 6.1: Start Your Engines!

Name	Due Date
Mr. McKenney's science classes were creating simple toy rocket successful groups are shown below, where h is the height in yar Group A: $h = -4t^2 + 8t + 12$ Group B: $h = -2t^2 + 4t + 16$	is using different accelerants. The equations that represents the two most indicate a solution of the ground after t seconds:
Groups were scored using the following rubric:	
Must be launched from a height of at least 10 feet.	10 pts
Must stay in the air for at least 4 seconds.	10 pts
Must reach a height of no less than 15 feet.	15 pts
Must reach its highest height in less than 2 seconds.	15 pts

1. For Group A, find the intercepts and vertex and then explain how they would have been scored on the rubric. Include your work. Group A: $h = -4t^2 + 8t + 12$

Group A's vertex:

Group A's x-intercepts:

Group A's y-intercept:

Group A's rubric scores Score	Explanation of Evidence
а.	
b.	
С.	
d.	

2. For Group B, find the intercepts and vertex and then explain how they would have been scored on the rubric. Include your work. Group B: $h = -2t^2 + 4t + 16$

Group B's vertex:

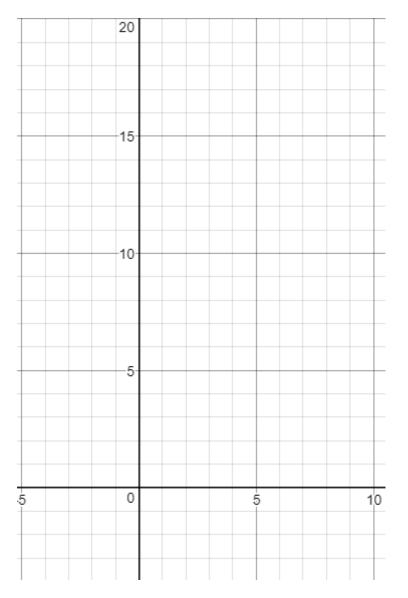
Group B's x-intercepts:

Group B's y-intercept:

Group B's rubric scores Score a.	Explanation of Evidence
b.	
С.	
d.	

3. Which group had the highest score? _____

4. Sketch the graphs of both groups. Sketch Group A's in one color of colored pencil and label it. Sketch Group B's in another color and label it.



Handout 6.1: Start Your Engines! - Key

Mr. McKenney's science classes were creating simple toy rockets using different accelerants. The equations that represents the two most successful groups are shown below, where **h** is the height in yards above the ground after **t** seconds:

Group A: *h* = -4*t*² + 8*t* +12

Group B: $h = -2t^2 + 4t + 16$

Group A's rubric scores

Groups were scored using the following rubric:

a.	Must be launched from a height of at least 10 feet.	10 pts
b.	Must stay in the air for at least 4 seconds.	10 pts
c.	Must reach a height of no less than 15 feet.	15 pts
d.	Must reach its highest height in less than 2 seconds.	15 pts

1. For Group A, find the intercepts and vertex and then explain how they would have been scored on the rubric. Include your work. Group A: $h = -4t^2 + 8t + 12$

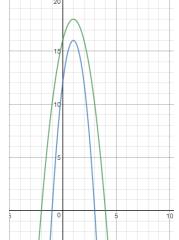
(1, 16)
(-1, 0) and (3, 0)
(0, 12)

Oloup A stub		
	Score	Explanation of Evidence
a.	10	The y-intercept is 12 so the launch point is 12 feet above the ground.
b.	0	The x-intercepts are -1 and 3 so it hit the ground in only 3 seconds.
с.	15	The vertex was (1,16) so the highest point was 16 feet above ground.
d.	15	The vertex was (1, 16) so it reached its highest point in 1 second.

2. For Group B, find the intercepts and vertex and then explain how they would have been scored on the rubric. Include your work. Group B: $h = -2t^2 + 4t + 16$

Group) B's vertex:	(1, 18)		
Group B's x-intercepts:		(-2,0) and (0, 4)		
Group B's y-intercept:		(0, 16)		
Group A's rub	ric scores			
·	Score	Explanation of Evidence		
а	10	The y-intercept is 16 so the launch point is 16 feet above the ground.		
b.	10	The x-intercepts are - 2 and 4 so it stayed in the ground for 4 seconds.		
с.	15	The vertex was (1,18) so the highest point was 18 feet above ground.		
d.	15	The vertex was (1, 18) so it reached its highest point in 1 second.		
		20		

3. Which group had the highest score? __Group B_____
4. Sketch the graphs of both groups
Sketch Group A's in one color of colored pencil and label it. Sketch Group B's in another color and label it.



Handout 6.2: Performance Task Rubric

Level	Mastery Level	Finds Group A's vertex, x-intercepts, and y-intercepts	Reports Group A's scores and explains evidence	Finds Group B's vertex, x-intercepts, and y-intercept	Reports Group B's scores and explains evidence	Sketches and labels both groups' graphs correctly.
4	Exemplifying Mastery	Finds all 4 key features correctly	Reports all 4 scores correctly with evidence.	Finds all 4 key features correctly	Reports all 4 scores correctly with evidence.	Sketches graphs of both groups and labels both correctly.
3	Approaching Mastery	Finds 3 of the 4 key features correctly.	Reports 3 scores correctly with evidence.	Finds 3 of the 4 key features correctly.	Reports 3 scores correctly with evidence.	Sketches and labels both graphs but has one error.
2	Developing Mastery	Finds 2 of the 4 key features correctly.	Reports 2 scores correctly with evidence.	Finds 2 of the 4 key features correctly.	Reports 2 scores correctly with evidence.	Sketches and labels both graphs but has two errors.
1	Not Representing Mastery	Finds 1 of the 4 features correctly.	Reports 1 score correctly with evidence.	Finds 1 of the 4 features correctly.	Reports 1 score correctly with evidence.	Sketches and labels both graphs but has 3 or more errors.
0	No Evidence of Mastery	Does not find any of the 4 features correctly.	Reports 0 scores correctly with evidence.	Does not find any of the 4 features correctly.	Reports 0 scores correctly with evidence.	Sketches are missing or show more than 3 errors.

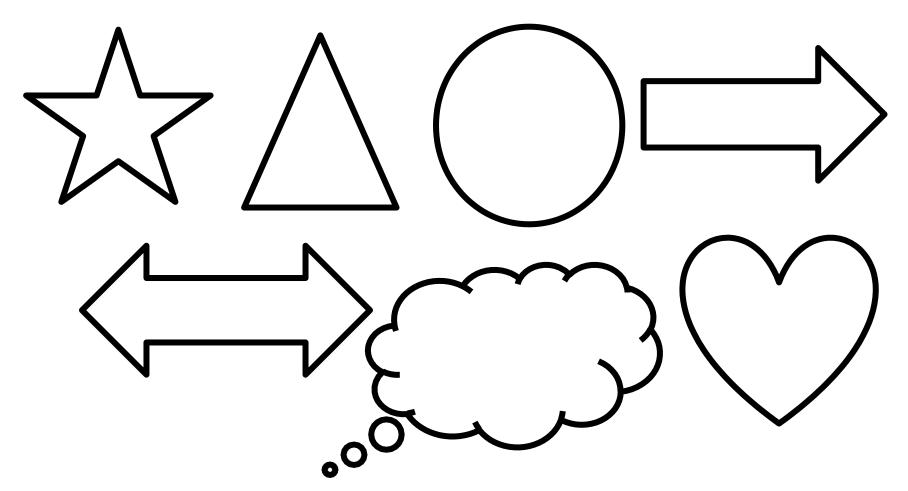
Handout 6.3: Fun Shape Unit Summary

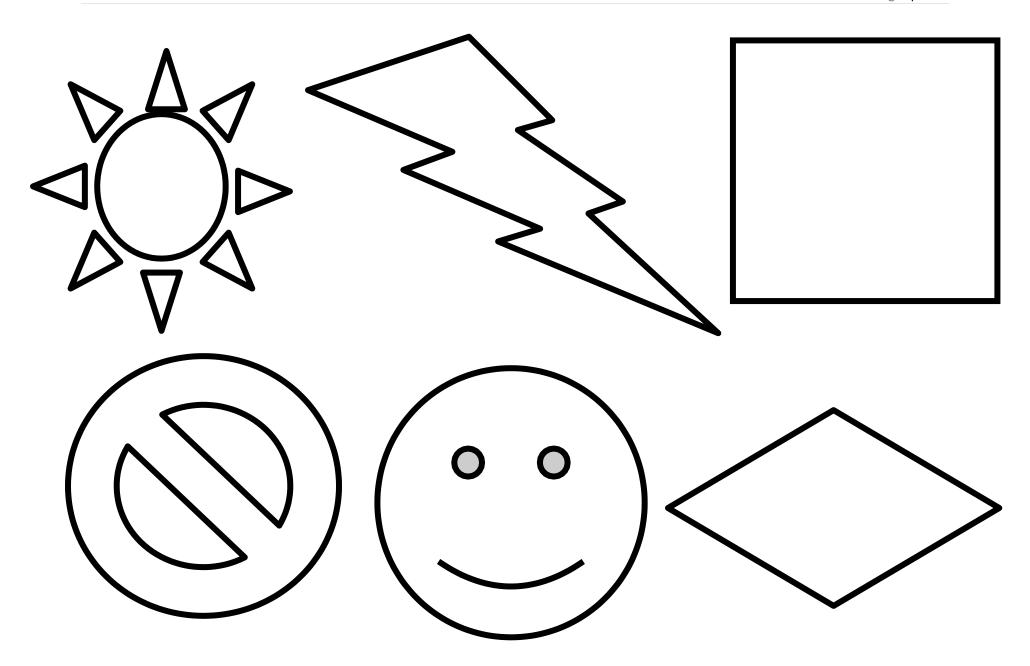
Name _____

Due Date_____

Directions:

- 1. Select and cut out 5 shapes that tell your progression of learning over this unit.
- 2. You may use cardstock/color paper, colored pencils/crayons, and other items to decorate your final product.
- 3. Shapes can be used multiple times/duplicated.







For training or questions regarding this unit, please contact:

exemplarunit@mdek12.org